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SSM JOURNAL

editorial

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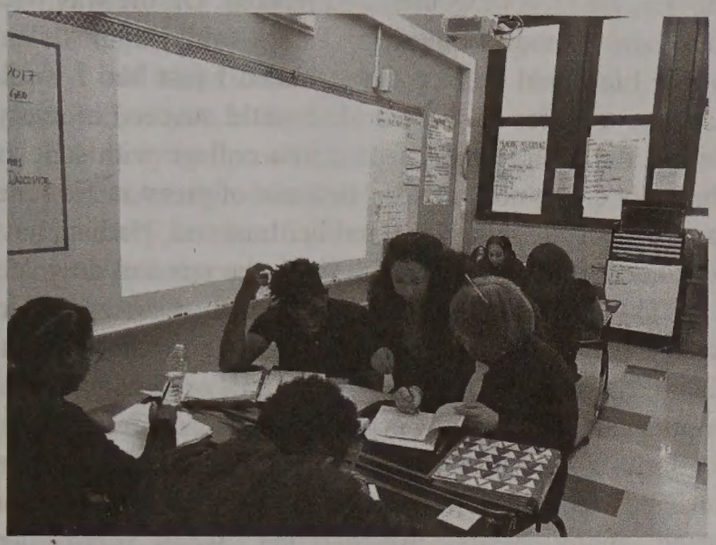
Dear *School Science and Mathematics* Journal Readers



In my effort to help "Fight the Fight for Public School Teachers," I have written annual editorials about teachers or teacher educators: Cindy Perdue, who taught kindergarten for 38 years in New Castle, Indiana; Maureen Reichardt, who after 33 years as a public school teacher resigned her position and is now at Guerin Catholic High School in Noblesville, Indiana; and Dr. Beatriz D'Ambrosio, a former mathematics teacher educator at Miami University in Oxford, Ohio, who worked to honor and support public school teachers. This will be the fourth such editorial.

Please let me introduce you to Halla Shteiwi. Halla teaches mathematics and is the Mathematics Department Head at Hughes STEM High School (HSHS), located diagonally across the street from my office at the University of Cincinnati (UC). [Full disclosure: Ms. Shteiwi was a student in courses I taught at UC.] I interviewed Halla at HSHS in her classroom after school and audio-recorded the conversation (November 13, 2017).

When asked, "Why did you want to be a teacher?" Halla explained. Her father wanted her to work in his business so, therefore, business was her first major but she "wasn't learning anything." Her next major was law, which required "too much reading." Not at all sure about where to turn next, she began studying in the Math Lab at UC and at times, helping other students with their mathematics homework. The person in charge of the Math Lab asked her if she wanted to work there and his question made her think: "Maybe I should be a math teacher." Before a final decision on her major, she took one education course and was assigned a field experience. While in the field experience classroom, she was supposed to just observe but that seemed impossible when so many students needed help. She broke with protocol and helped the student sitting at the desk next to hers and realized: "This is for me!"

That was the beginning of her quest to become a teacher. Ms. Shteiwi earned dual UC degrees in December 2009, a B.S. in Secondary Education, and a B.S. in Mathematics. During the summer of 2010, she took a trip to her parents' birthplace, Jordan, and did not return before the start of the 2010–2011 academic year. In October of 2011, I heard that an HSHS teacher had quit and suggested Ms. Shteiwi as a possible substitute. Ultimately, she was hired and began to teach in January of 2011. Halla described her first semester at HSHS as difficult and stressful. Accordingly, the students had "no manners." This was not how she was raised. She recalled screaming at them and even telling them to shut up. Yet, she ultimately learned to change her tone of voice, ask questions rather than offer advice, and "never yell at a kid." Giving credit to her colleagues, Mr. Lane, Ms. Vagedes, and Mr. Frecker, for these lessons, Halla said she "continued to grow." Running an effective classroom is not an issue for Halla now, but she was quick to say, "They [the students] do not like me at the beginning of the year and they learn to love me at the end."

With classroom discipline no longer an issue, Halla focused on establishing her teacher identity. In fact, one aspect of her identity is her belief that *all* students can learn and her role is to make that happen. I asked her, "Why do you have a stance that *all* kids can learn?" Halla's succinct answer: "My background." Halla's parents were "extremely poor" when they immigrated to America from Jordan. Her father completed the equivalent of a sixth-grade education and her mother completed the

equivalent of a 10th-grade education. Rakan Shteiwi worked three different jobs with the commitment that his children were not "going to be like me." Serendipitously, while working as a waiter Rakan became friendly with a banker, who loaned him money to start a restaurant. Using his first business profits from this restaurant, Mr. Shteiwi paid the loan back and opened more restaurants. Halla said her father instilled these things in her: "If you are sick you are still going to go to school unless you are throwing up or bleeding.... Do everything you need to be on the honor roll.... No after-school activities unless you maintain your grades." She recalled a time when she asked for a calculator and her father said, "Absolutely not." Halla and her siblings were required to learn English and Arabic. Although her father worked every day of his life, he ritually came home in the evenings during homework time to help his children with Arabic and school homework. When Halla was 15, she started working in the family-owned restaurant and continued to work there, either full-time or part-time, for the next 10 years. Work ethic and a "no excuse" philosophy instilled by her father are reasons Halla believes *all* students can learn. Failure is not an option.

Let me interrupt Halla's story with my observations. I would describe Ms. Shteiwi as a *warm demander* (Bondy, Ross, Hambacher, & Acosta, 2008). The description *warm demander* first appeared in Kleinfeld's (1975) study when he used it to portray the ways in which effective teachers of Eskimo and Alaskan Indian children interacted with their students. Warm demanders "embrace values and enact practices that are central to their students' success" (Bondy et al., 2008, p. 420). Bondy and Ross (2012) encapsulated a warm demander teacher as one who treats "students with unconditional positive regard, knowing students and their cultures well, and insisting the students perform to a high standard" (p. 58). Ware (2006) used two terms to describe a warm demander: caregiver and authority figure.

Like Halla's father, warm demanders establish a "no excuses" approach (Wilson & Corbett, 2001). They will not accept students' excuses for failure, but more critically, they blame themselves if their students fail. Bondy and Ross (2012) described practices that warm demanders enact: "Build Relationships Deliberately; Learn About Students' Cultures; Communicate an Expectation of Success; [Move] Beyond Believing to Insisting; Provide Learning Supports; Support Positive Behavior; and, Be Clear and Consistent with Expectations" (pp. 2–6). (Please read the entire article for more details of each of these practices.)

Let me return to Halla's story by retelling one of her stories of a former student, Tamra (pseudonym), who is now about 21 years old. When Tamra was a student in Halla's mathematics classroom, Halla found out that Tamra's home life was in shambles. Tamra was taking care of her siblings because her mother moved to another city in another state. Halla told Tamra, "If you need time away you can always come with me." Halla got parent consent and took Tamra to Halla's parents' home for a three-day weekend. They played family games, watched movies, ate meals together, and relaxed. On the way back to school Monday morning, Tamra told Halla: "If my whole life could be like the weekend I just had I would have no troubles in the world. I could succeed at everything I did." She was accepted into college with scholarships but chose not to attend because of pressure from her mother. Despite this, Halla and her husband, Nathan, have continued to mentor Tamra through the ups and downs of life. They recently helped Tamra furnish an apartment with a "couch, vacuum cleaner, dining room table and chairs, dishes" and other items. Tamra originally wanted to pursue a career in law enforcement but is now working for an in-home health care company. Ms. Shteiwi and her husband built a strong relationship with Tamra that has lasted many years. Recall that warm demanders "deliberately build relationships" with students (Bondy & Ross, 2012).

The perception that relationships end when students leave our classrooms is a myth for Halla (and many public school teachers!). Teaching days are short and teachers have summers off are also myths for teachers like Halla. On a typical workday "since baby" (darling Laila is not quite two years old), Halla rises at 5:30 a.m. and arrives at HSHS between 7:00 and 7:30 a.m., this while sometimes "acting out the lesson ... thinking of questions to ask students ... while driving." Before students arrive, she sets up technology or other equipment, assigns students to groups, and arranges the desks for the group work. During her planning bell she writes lesson plans with her student teacher. As added duty because of her role as Mathematics Department Head, Halla observes and evaluates mathematics teachers in her department. She also does so during her planning bell.

School ends at 3:25 p.m. but the work does not stop then. On Monday, Tuesday, and Wednesday she stays late to grade papers, review exit tickets, schedule tutors, and make Show Me™ apps for days she needs a substitute teacher, just to name a few tasks. When asked about how many students she works with each day she told me, "134 students on Monday, Tuesday, and Friday. 96 students on

Wednesday and about 43 students on Thursdays." On Thursdays students do iWorks™ which is an online tutorial system; they work on individualized mathematics skills that Halla assigns each week. All of this while Halla is "constantly thinking, How can I make it better?"

So, back to me. If I feel isolated in my so-called "ivory tower" all I need to do is walk across the street and Halla welcomes me into her classroom. One time I when I was visiting another teacher she saw me in the hallway and asked me to come into her class. She introduced me to her students: "This is Dr. Harkness. She was my teacher." Well, if you have ever been a teacher you can appreciate how I felt when I was introduced as a teacher, not a professor mind you, a teacher.

This story has provided a glimpse of the life of one public school teacher, Ms. Shteiwi.

Her story is one of *warm demanding*. Care. Authority. Not accepting excuses. Having high expectations. Building relationships. A stance that *all* students can learn. She learned some of these lessons from her father, Rakan Shteiwi, an immigrant from Jordan. She learned some lessons from dedicated colleagues. And in her own words, she is "still growing" as a mathematics teacher in an urban public school.

To end, let me quote Diane Ravitch (2013), former assistant U.S. Secretary of Education:

> Public policy should leverage federal and state funding to reduce poverty and racial isolation in schools . . .preserving public education and strengthening it rather than privatizing it is critically important to preserve our democracy. And blaming and punishing the people who work in our neediest public schools for the persistence of social, economic, and educational inequities is not just wrong, but will further damage the children in whose interest the corporate reformers claim to be acting. We do not help children by demoralizing *those who do the most difficult work* with them under the most difficult conditions, *every day*. (p. 299; italics added for emphasis)

Children need teachers like Ms. Shteiwi. I was her teacher and she has been mine.

Warm Regards,

Shelly Sheats Harkness
University of Cincinnati

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Enhancing Formative Assessment Practice and Encouraging Middle School Mathematics Engagement and Persistence

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In the transition to middle school, and during the middle school years, students' motivation for mathematics tends to decline from what it was during elementary school. Formative assessment strategies in mathematics can help support motivation by building confidence for challenging tasks. In this study, the authors developed and piloted a professional development program, Learning to Use Formative Assessment in Mathematics with the Assessment Work Sample Method (AWSM) to build middle school math teachers' understanding of the characteristics of high-quality formative assessment processes and increases their ability to use them in their classrooms. AWSM proved to be feasible to implement in the middle school setting. It improved teachers' practice of formative assessment, especially in their feedback practices, regardless of their pedagogical content knowledge at entry. Results from focus groups suggested that teachers were better able to implement ungraded practice and student self- and peer-assessment after AWSM, and that students were more willing to engage in complex problem solving.

In the transition to middle school, and during the middle school years, students' motivation for mathematics tends to decline from what it was during elementary school. At this age, students report less valuing of mathematics and lower effort and persistence in mathematics problem solving over time (Pajares & Graham, 1999; Valas, 2001; Wigfield, Eccles, & Pintrich, 1996). Middle school students also report lower confidence in their mathematics ability than before (Clarke, Roche, Cheeseman, & van der Schans, 2014; Pintrich & Schunk, 1996), influenced in part by exposure to a larger peer group with whom they begin to compare themselves, more perceived competition in the classroom environment, and more rigorous standards for evaluation (Eccles & Midgley, 1989). Some students perceive that they are judged more on innate ability in middle school than on improvement (Anderman & Midgley, 1997), and come to believe that if they lack ability, their effort will not help (Valas, 2001). Students also lose confidence that they can self-regulate, in that they feel less able to organize their thoughts, understand complex tasks, and choose and evaluate problem-solving strategies (Pajares & Graham, 1999).

Low Confidence in Mathematics Affects Teaching and Learning

These feelings have an effect on teaching and learning in the middle school mathematics classroom. Students who are not confident that they can solve complex problems, or who do not see the point of putting forth effort to do so, try to avoid those tasks or pressure teachers to make the work simpler for them (Clarke et al., 2014). This lack of mathematics self-efficacy is a predictor of lower mathematics achievement outcomes (Pajares & Graham, 1999). Some middle schoolers will attempt to engage in mathematics learning only when tangible rewards are offered (Rowan-Kenyon, Swan, & Creager, 2012). Students' difficulties with motivation and persistence, therefore, become a problem of practice for middle school teachers.

In this environment, middle school mathematics teachers can feel hesitant giving students challenging and complex work. If they do so anyway and students encounter difficulty, some teachers oversimplify the task or tell students how to solve it (Clarke et al., 2014; Ferguson, 2009). This is especially true when teachers work with lower-achieving students (Zohar, Degani, & Vaaknin, 2001). However, the Common Core and other contemporary U.S. mathematics standards require that students be able to solve complex problems and to explain their reasoning, so teachers need effective instructional strategies to support students in these practices.

Instruction Can Build Confidence for Challenging Mathematics Tasks

Research has supported some features of instruction and the classroom environment that make it easier for middle school mathematics teachers to pose challenging tasks and for students to engage, persist, and succeed in them. Rather than lower the level of cognitive demand of difficult problems, teachers have found success by supporting

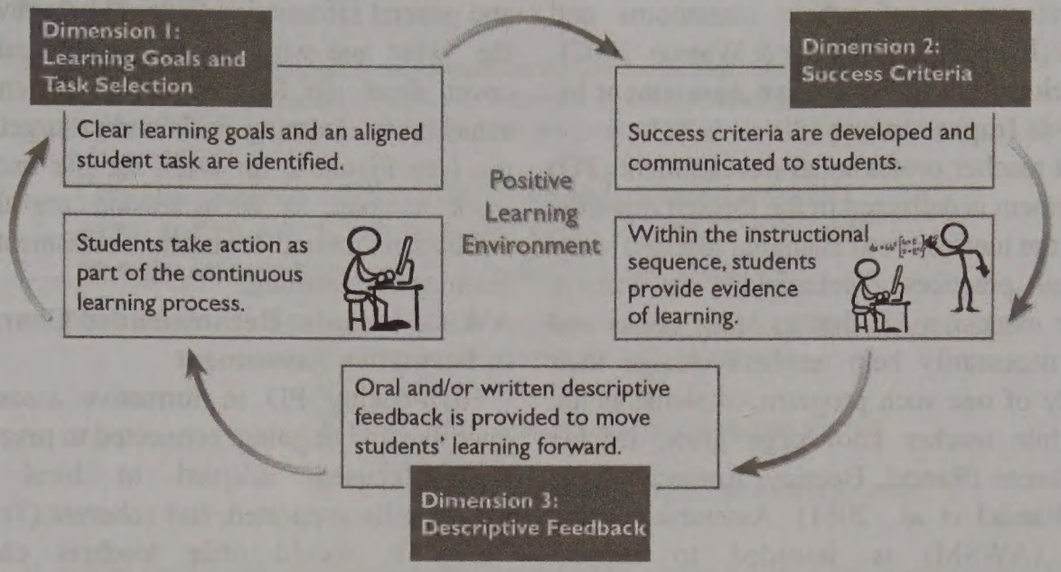
Dimension 1: Learning Goals and Task Selection
Clear learning goals and an aligned student task are identified.
Dimension 2: Success Criteria
Success criteria are developed and communicated to students.
Positive Learning Environment
Students take action as part of the continuous learning process.
Within the instructional sequence, students provide evidence of learning.
Oral and/or written descriptive feedback is provided to move students' learning forward.
Dimension 3: Descriptive Feedback

Figure 1. AWSM formative assessment process.

small-group discussions and peer collaboration (Ferguson, 2009; Rowan-Kenyon et al., 2012; Ryan & Patrick, 2001; Schunk & Pajares, 2001; Zohar et al., 2001). When teachers demonstrated to students that they are supportive and will try to help students as individuals, and when teachers had clear expectations that students would respect one another's ideas in the classroom, students were less disruptive and better able to self-regulate (Ryan & Patrick, 2001; Valas, 2001). A classroom focused more on mastery and improvement rather than competition also supported students' academic confidence (Ryan & Patrick, 2001).

Formative Assessment Strategies Can Help Students Be More Confident Mathematics Learners

Formative assessment is an evidence-based process of gathering information on three questions: (a) Where am I going? (b) How am I doing now? (c) Where do I go next? to support a learning cycle (Hattie & Timperley, 2007; Sadler, 1989). Therefore, the most important formative assessment practices involve (a) students' understanding of their learning goals and targets, (b) the criteria by which they will know how they are progressing with their learning, and (c) what needs to be done next to move learning forward. Feedback is an active part of the process and can address the task, the student's processing of the task, suggestions for what to work on next, and scaffolds for the individual student (Hattie & Timperley, 2007). Literature supports prioritizing these three dimensions of formative assessment (see Figure 1).

Assessment-centered teaching can have profound effects on student learning and motivation (DiRanna et al., 2008). Well-designed formative assessment is associated with major gains in student achievement across all ages and subjects, and has its greatest positive impact on students who struggle in mathematics (Black & Wiliam, 1998a, 1998b). To understand learning targets, students need to have clear knowledge of where they are going with their learning and avoid the inefficiency and frustration of trial and error (Sadler, 1989). Students who struggle in school particularly benefit from understanding learning targets, because they do not intuit the targets on their own (Black & Wiliam, 1998a, 1998b; James et al., 2006). Clear success criteria enable students to understand how they are doing on learning targets, so they can judge and self-regulate the quality of their work (e.g., Andrade, Du, & Mycek, 2010; Brookhart, Andolina, Zusa, & Furman, 2004; Sadler, 1989). When teachers involve students in the assessment process, students perceive more control of and more responsibility for their own learning (Rieg, 2007). Allowing students to help generate success criteria gives them a feeling of empowerment and makes assessment of their work seem less punitive and more constructive (Brookhart, 1997). In turn, students feel more competent and are more likely to engage in learning (Stiggins, Arter, Chappuis, & Chappuis, 2006).

Formative Assessment Is Particularly Difficult in Mathematics Without Support

Formative assessment requires teachers to elicit students' existing ideas as students make their thinking visible to help further understanding. In mathematics, it is often difficult to interpret students' understanding, knowledge, and learning of complex content. Interpretations of student work as evidence of learning may be powerfully influenced by teachers' own understandings of mathematics concepts and how the concepts can be communicated, and expectations

regarding the performance of whole classrooms and individual students (Even, 2005; Morgan & Watson, 2002).

Professional Development in Formative Assessment in Mathematics Needs Improvement

Much of current teacher professional development (PD) in formative assessment is delivered in the form of one-time workshops, which are ineffective at changing teachers' day-to-day professional practices (McLaughlin & Talbert, 2006). Even more extensive PD that includes books and videos does not necessarily help teachers change their practice. In a study of one such program, implementation was low, and while teacher knowledge grew, teacher practice did not change (Randel, Beesley, Apthorp, Clark, & Wang, 2016; Randel et al., 2011). Assessment work sample method (AWSM) is intended to support mathematics teachers in using, not just learning about, high-quality formative assessment strategies linked to the overall formative assessment process.

Program

Learning to Use Formative Assessment in Mathematics with the AWSM is a PD program that builds middle school mathematics teachers' understanding of the characteristics of high-quality formative assessment processes and increases their ability to use them in their classrooms. We were inspired to create AWSM following the results from previous research (Randel et al., 2011, 2016) on a formative assessment PD program developed for all content areas and levels of learning. This program did not change teacher practice in mathematics, in part because it had few examples of how to implement formative assessment in that content area. By contrast, AWSM provides PD that builds formative assessment practices and skills specifically in mathematics. AWSM is about helping teachers develop effective instructional practices embedded in everyday teacher lessons and uses actual student and teacher work along with peer review/support.

In AWSM, reviewing and discussing samples of student work help teachers shift from thinking of teaching as something teachers do to a focus on learning as something students do, because it is in student work that student thinking is made visible (e.g., Hattie, 2009). The AWSM approach uses the *Assessment Work Sample Method*, a way to collect, create, discuss, and learn about a formative assessment process using student work samples that help ground the learning in daily practice (Clare, Valdés, Pascal, & Steinberg, 2001; Matsumura et al., 2006). The work samples include a teacher cover sheet that conveys the goals of the lesson, the type of knowledge/skill being developed, the success criteria for meeting learning goals, and general information that will help reviewers understand the "what and why" of the assignment. Attached to the cover sheet are four pieces of student work, two that achieved the learning goals and/or targets and two that did not (see Figure 2 for work sample excerpt). In AWSM, work samples in each session are the foundation of discussions around formative assessment and mathematics teaching and learning.

AWSM Includes Recommended Characteristics of PD in Formative Assessment

High-quality PD in formative assessment should be intensive and ongoing, connected to practice, collaborative, content-focused, adapted to local context, active, systemically supported, and coherent (Trumbull & Gerzon, 2013). It should offer teachers choices about the instructional practices they will focus on, the flexibility to adapt formative assessment to their context, provide teachers with a lot of time to change practices, and offer both support and accountability from peers (Leahy & Wiliam, 2012). AWSM has these recommended features, as well as a content focus on mathematics. Specifically, AWSM places teachers in collaborative learning groups, features ongoing meetings throughout the year, and uses work samples to connect the PD to content, instructional practice, and local context. The sessions aim to prompt teachers to examine their current assumptions and decide which aspects of their practice to change, provide time to learn new approaches, and stimulate reflection that leads to strategy implementation.

AWSM is structured around nine face-to-face meetings which include a two-day introductory workshop and eight sessions of about 45 minutes each. The introductory workshop is designed to help participants gain a shared understanding of the formative assessment process and prepare to create positive classroom environments while the eight short sessions are designed to support teachers as they implement the process. During Part One of the introductory workshop, participants build their understanding of formative assessment as one component of a larger assessment system, with an emphasis on providing descriptive feedback to move the learning forward. Participants discuss the characteristics of positive classroom culture and why it is essential, and consider structures that will help them create such a culture. Participants learn about fixed and growth-oriented mindsets (Dweck, 2006) and the type of feedback that will promote a growth-oriented mindset that builds student self-efficacy for mathematics. They discuss physical, social, and emotional factors that impact classroom culture and share successes and challenges of creating such cultures in their classrooms.

Teacher Cover Sheet

5. What were your learning goals for the students for this assignment? In other words, what skills , concepts, or facts did you want students to learn, practice or demonstrate understanding of as a result of completing this assignment? (Students will know and understand that: ...)

I wanted students to demonstrate an understanding of unit rates and be able to calculate unit rates fluently ... or be able to make comparisons when the rates were not the same.

6. Check the type of learning goal/target this assignment addresses (check all that apply):

- ☑ Knowledge (facts/details to be memorized)
- ☑ Skill (algorithmic procedures)
- ☑ Conceptual Understanding (reasoning, generalizing, explaining, etc.)
- ☑ Problem Solving within a Context (multiple procedures; solution strategy)

11. a. How was this assignment assessed? If there is a rubric, student reflection, etc., please attach it. If you are not attaching a rubric, please explain your criteria for determining if students met the learning goal of the assignment.

The rubric is attached (on back of assignment).

11. b. Did you share these criteria with the students? ☑ Yes [ ] No

Student Work

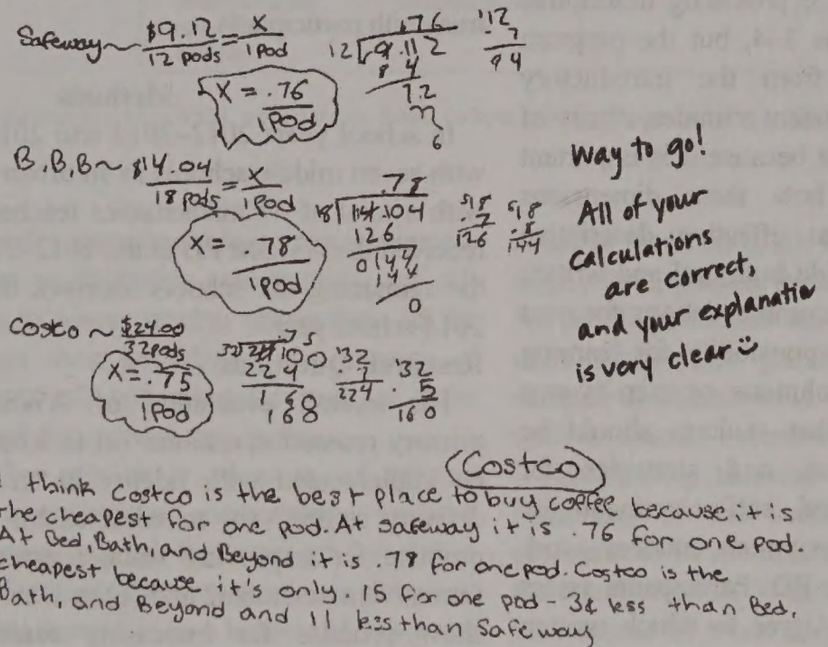
Safeway ~ $9.12 = X / 12 pods = 1 pod
X = .76 / 1 pod
B,B,B ~ $14.04 = X / 18 pods = 1 pod
X = .78 / 1 pod
Costco ~ $24.00 / 32 pods
X = .75 / 1 pod
Way to go!
All of your calculations are correct, and your explanation is very clear ☺
I think Costco (Costco) is the best place to buy coffee because it is the cheapest for one pod. At Safeway it is .76 for one pod. At Bed, Bath, and Beyond it is .78 for one pod. Costco is the cheapest because it's only .15 for one pod - 3¢ less than Bed, Bath, and Beyond and 11 less than Safeway

Figure 2. AWSM work sample.

In Part Two of the introductory workshop, participants plan for implementing formative assessment by first examining authentic student work. In collaborative groups, teachers analyze the intended learning goals and success criteria from anonymous work samples and then compare the teachers' intended learning goals to accompanying student work. Using criteria, participants determine if the teacher's intended learning and the actual student tasks are strongly aligned, partially aligned, or weakly aligned. Mathematical content as well as the inferred cognitive demand of both the learning goal and student task are reviewed and discussed. Through this analysis, participants clarify their understanding of Dimension 1 (learning goal and aligned student task) and Dimension 2 (success criteria

aligned to the learning goal and student task). These dimensions are the foundation for the formative assessment process because without clarity about what is to be learned and clear criteria for goal attainment, the feedback process can be derailed and learning impeded. When these dimensions are clear, students are supported in taking greater ownership for learning.

During Part Three of the introductory workshop, participants apply the learning from Parts One and Two to their own instructional planning. During this segment, participants analyze an instructional unit they plan to implement to determine whether the learning goals, student tasks, and success criteria are strongly aligned. Participants consider strategies for how to best communicate learning expectations to students and identify/adapt tools to help students track their own progress. Independent work time allows participants to initiate revisions to their instructional unit if needed and receive feedback from peers and workshop facilitators.

The short sessions are scheduled around grade level team times and are organized as teacher learning communities with a facilitator that has both mathematics and formative assessment expertise. Dimension 3, providing descriptive feedback, is the focus of Sessions 1–4, but the program continues to reference topics from the introductory workshop such as class culture, student attitudes, clarity of learning goals, and success criteria because it is important for participants to understand how these dimensions intersect. Participants learn what effective descriptive feedback entails and how to provide both oral and written feedback. Cues, questions, and recommendations for next steps help students take more responsibility for learning than simply providing correct solutions or step-by-step actions. Participants also learn that students should be active in the feedback process, and strategies for implementing effective peer- and self-assessment are discussed. In Sessions 1–5, anonymous student work samples remain at the core of the PD. Participants assess work samples to determine the degree to which written teacher comments will move student learning forward. During this process participants make clear connections between AWSM Dimensions 1–3, as providing effective feedback is dependent on clearly articulated learning goals and success criteria. Participants examine work samples that incorporate student peer and self-assessment, and they use the AWSM rubric (adapted from Matsumura, Garnier, Pascal, & Valdes, 2002) to independently assess anonymous work samples and practice providing written feedback.

Rather than analyze the student work of an unknown teacher, Sessions 6–8 ask participants to share their instructional practice more directly. In these sessions, participants generate their own student work samples, present the work to colleagues, and request feedback on how to improve their own implementation of the process. These sessions are powerful as the entire formative assessment process is fully integrated. Participants reflect on their own progress for implementing formative assessment and identify next steps. The formative assessment process is modeled during the each of the sessions. Clear goals and success criteria are provided, procedures are modeled and discussed, and feedback is incorporated to support learning among the participants. Clarifying this approach and calling out the modeling helps participants think about what their students need from them to engage in an ongoing feedback process.

In AWSM, the expertise of the session facilitator should not be underestimated. The mathematics teachers bring targeted questions to these sessions, and the facilitator must be prepared to reference research, share how they themselves were able overcome challenges with using formative assessment, and continue to build and maintain trust with participants.

Methods

In school years 2012–2013 and 2013–2014, we worked with seven middle schools in an urban district in Colorado, with a total of 47 mathematics teachers. One pilot school received the AWSM PD in the 2012–2013 school year, and the remaining six schools received the PD in the 2013–2014 school year.

Research Questions

The overall evaluation of AWSM addresses three primary research questions: (a) to what extent can AWSM be implemented with fidelity in an authentic education delivery setting? (b) to what extent does AWSM show promise for improving teacher practice of mathematics formative assessment? and (c) to what extent does AWSM show promise for increasing student achievement in mathematics? The research involved teacher measures of formative assessment practice, student measures of mathematics achievement, observations of professional development sessions, and teacher questionnaires and focus groups.

Data Collection

To answer the aforementioned research questions, data were collected from (a) teacher work samples for assessing teacher practice in formative assessment, (b) a pretest of content and pedagogical knowledge in mathematics, (c) a

	4	3	2	1
Focus of the goals on student learning	Goals are **highly focused** on student learning. Goals are: ▪ fully elaborated (all aspects and parts of the learning goal are included and described); • appropriate to student learning needs (grade/course appropriate); and • clearly communicated to students.	Goals are **focused** on student learning. Learning goals: ▪ are clear statements (not an activity, topic, phrase, or question); ▪ are appropriate to student learning needs (grade/course appropriate); and/or ▪ may or may not be clearly communicated to students.	Goals are **somewhat focused** on student learning. Goals: ▪ are stated as activities or questions with no clear description of what students should know or be able to do; • may not be appropriate to student learning needs (grade/course appropriate); or • may not be clearly communicated to students.	**No learning goals** are provided for the task. ▪ no clear description of what students should know or be able to do; or ▪ topic rather than learning goal may be listed.
Clarity of the student assessment criteria	Assessment criteria are: • communicated so that students clearly understand the assessment criteria (with detailed elaborations) for high quality, medium quality, and low quality work; • made explicit through oral dialog, written criteria, and/or leveled examples of student work; and • discussed with students.	Assessment criteria are: ▪ communicated so that students understand the assessment criteria (but without detailed elaborations) for high quality, medium quality, and low quality work; and • communicated through at least written instructions but may also provide examples and oral instructions.	Assessment criteria are: ▪ communicated so that students know what they must do, but the criteria do not delineate between what high quality, medium quality, or low quality work involves (e.g., may include a checklist or rubric based on presence or absence of work elements).	No assessment criteria are stated or it is not possible to determine the assessment criteria from the work sample documents.

Figure 3. Sample of two dimensions of AWSM work sample rubric (adapted from Matsumura et al., 2002). Higher scores indicate higher formative assessment quality.

district-wide mathematics assessment to assess the impact of AWSM on student mathematics achievement, and (d) teacher focus groups to assess teacher perceptions of the impact of AWSM on their knowledge and practice of formative assessment. Data collection for each of these outcomes measures are discussed in turn below.

Work samples. To provide a measure of teacher practice of formative assessment, participating teachers provided samples of student work that were graded and/or marked with teacher feedback. Student work samples were linked to an assignment that could be a homework/seatwork assignment, performance assessment, or quiz/in-class assessment. For each assignment, teachers completed a cover sheet to describe the assignment (including directions given to students); the source of the assignment (e.g., teacher created; from a textbook, workbook, curriculum publisher, etc.); why the particular assignment was selected; the purpose of the assignment (e.g., check student progress; provide practice, unit post-test); the student learning goals for the assignment; student accommodations for the assignment (e.g., addressing different skill levels; whether students received help); how the assignment was assessed (e.g., rubric; grading system; student reflection); and criteria for deciding whether students met learning objectives. For each assignment, participating teachers provided four samples of student work—two that met learning objectives and two that did not meet learning objectives.

Participating teachers submitted work samples at the beginning of the school year, prior to the AWSM professional development sessions (pretest), and again at the conclusion of AWSM program (post-test). Two analysts both scored all work samples using the AWSM rubric (see Figure 3 for sample) on a scale from 1 to 4, indicating the level of quality of formative assessment practices on six separate dimensions: (a) focus of the goals on student learning; (b) alignment of learning goals and task; (c) alignment of learning goals and assessment criteria; (d) clarity of the student assessment criteria; (e) feedback type; and (f) feedback integrates student involvement. When scores for the two raters were discrepant, the raters discussed the work sample and reconciled their scores to produce one score per work sample per dimension.

Measure of pedagogical content knowledge. The learning mathematics for teaching (LMT) assessment (Learning Mathematics for Teaching Project, 2011) was the measure of teacher content and pedagogical knowledge in mathematics. The purpose of the LMT assessment is to capture teachers' mathematical knowledge for teaching (MKT). The LMT is not an assessment of "common" content knowledge alone, but knowledge that is specific and useful for teaching student mathematics. The assessment was designed to discriminate teachers' MKT across the entire ability range; therefore, items were chosen such that the average teacher answers 50% of them correctly. The developers stressed that the assessment not for drawing conclusions about the competency of individual teachers, but rather for purposes such as measuring the effectiveness of professional development. The LMT content area assessments included Middle School Number Concepts and Operations (NCO) and Middle School Patterns, Functions and Algebra (PFA), as these assessments were aligned to AWSM PD content.

Student achievement. For the impact analysis of the AWSM on student achievement, the outcomes were mathematics scores on the Measures of Academic Progress (MAP) assessment administered at two-time periods during the school year (fall and spring). We used a quasi-experimental difference-in-differences approach (Cook, Shadish, & Wong, 2008), comparing students of teachers who participated in the AWSM PD to their previous year's students prior to AWSM exposure. Two separate analyses were conducted for proximal and distal outcomes. The proximal outcome analysis was conducted at the end of the training year and included 7 schools, 47 teachers, 2,281 counterfactual group students, and 3,896 treatment group students. The distal outcome analysis was conducted one-year post-training and included 1 school, 3 teachers, 312 counterfactual group students, and 330 treatment group students.[1] Both the proximal and distal analyses utilized 2-level hierarchical linear models (HLM) to account for students nested within teachers. The proximal analysis included school dummy codes at level 2 to account for shared variance between students at the school level.

Teacher focus groups. Schools were to have two teacher focus groups in the fall 2013 semester and two in the 2014 spring semester. All but two schools had the four planned focus groups; due to scheduling difficulties two of the schools had three focus groups, resulting in a total of 22 focus groups for the 2013–2014 school year. Each focus group lasted for about 45 minutes. The focus groups were audio recorded and transcribed.

The semistructured focus group protocol addressed the research questions by asking for teachers' feedback about AWSM implementation ("How have the program facilitators supported you before and during AWSM?"), perceptions of impact on their classroom practice ("To what extent is AWSM building your ability to use formative assessment in your classroom?"), perceptions of impact on student learning ("What strategies seem most effective for student learning?"), and recommendations for improvement. Although researchers had the same protocol for all focus groups, the phase of AWSM implementation influenced which questions they asked at each group. For example, researchers did not ask about results from teachers' use of formative assessment strategies in the first focus group because it was too early then for teachers to have substantial experience of using the strategies.

Audio recordings of focus groups were transcribed, and the transcripts were imported into MAXQDA for analysis. Two coders developed initial codes based on the question topics in the focus group protocol. As coding progressed, the two coders conferred frequently on the coding scheme and the meaning of the codes, and collapsed some coding categories while expanding others into sub-codes. The final list of codes addressed session spacing and timing, fostering teacher collaboration, session mathematics content topics, facilitator support, understanding of formative assessment, need for professional development in formative assessment, implementation of formative assessment strategies, planning for formative assessment, formative assessment for differentiation, and results from formative assessment strategies. The emphasis of the analysis was to represent the points of view of the respondents, while also including some representative comments.

After each iteration of AWSM, the evaluators analyzed focus groups feedback and provided the developers with formative information to revise the PD materials. For example, the final version of AWSM included more content information in the introductory summer session and an earlier introduction of the work sample rubric, in response to teacher feedback.

Analysis and Results

Results below are provided for the teacher work samples, teacher pedagogical content knowledge assessment, student achievement, and focus groups.

Teacher Work Samples

Table 1 displays descriptive statistics by work sample dimension as well test statistics and effect sizes for comparisons of pre- and post-test work sample scores. Teachers showed the greatest improvement in feedback:

Table 1

Descriptive Statistics, Test Statistics, Effect Sizes for the Work Sample Pretest and Post-Test by Rubric Dimension (N = 47)

Rubric Dimension	Pretest Mean (*SD*)	Post-Test Mean (*SD*)	Test Statistic	*p*-Value	Cohen's *d*
Focus of the goals on student learning	2.23 (.70)	2.52 (.56)	.80	.43	.45
Alignment of learning goals and task	2.21 (.72)	2.32 (.57)	2.22	.03	.16
Alignment of learning goals and assessment criteria	1.38 (.49)	2.01 (.70)	5.02	.0001	1.03
Clarity of the student assessment criteria	1.43 (.50)	2.24 (.70)	6.54	<.0001	1.35
Feedback type	1.34 (.60)	2.53 (.60)	9.81	<.0001	1.98
Feedback integrates student involvement	1.17 (.38)	2.49 (.67)	12.48	<.0001	2.42
Average score across all dimensions	1.63 (.54)	2.35 (.61)	8.74	<.0001	1.79

"feedback integrates student involvement" ($d = 2.42$) and "feedback type" ($d = 1.98$). Participants' change scores also showed high effect sizes (greater than 1) on the two dimensions involving assessment criteria: "alignment of the learning goals and assessment criteria" ($d = 1.04$) and "clarity of the student assessment criteria" ($d = 1.35$). Participants showed the least improvement in the two dimensions involving learning goals: "focus of the goals on student learning" ($d = .16$) and "alignment of learning goals and task" ($d = .45$), as they scored relatively high at baseline. Scores did not differ significantly by teacher experience level.

Tests of statistical significance were conducted using HLM to account for nesting of teachers within school. Seven separate models were estimated for each of the work sample dimensions as well as the average score across dimensions. For the average work sample scores, the analysis revealed a significant increase from pre- to post-test (see Figure 4). For the analyses of individual dimensions, dimensions representing assessment criteria and feedback (Dimensions 3–6) showed statistically significant increases from pre- to post-test. For dimensions representing learning goals, change in "focus of learning goals on student learning" (Dimension 1) was statistically significant from pre- to post-test; however, change in "alignment of learning goals and task," while in the positive direction, was not statistically significant (see Table 1 for test statistics and *p*-values for each comparison). Overall, these findings suggest that the AWSM professional development program increased teachers' formative assessment practice, particularly in the areas of assessment criteria and feedback.

Pedagogical Content Knowledge

Baseline pedagogical content knowledge was to be used as a covariate in the student achievement model to assess the extent to which it affected the impact of the professional development on teacher practice and student achievement. Across both content areas, there was wide variability among teachers on assessment scores. Specifically, standardized

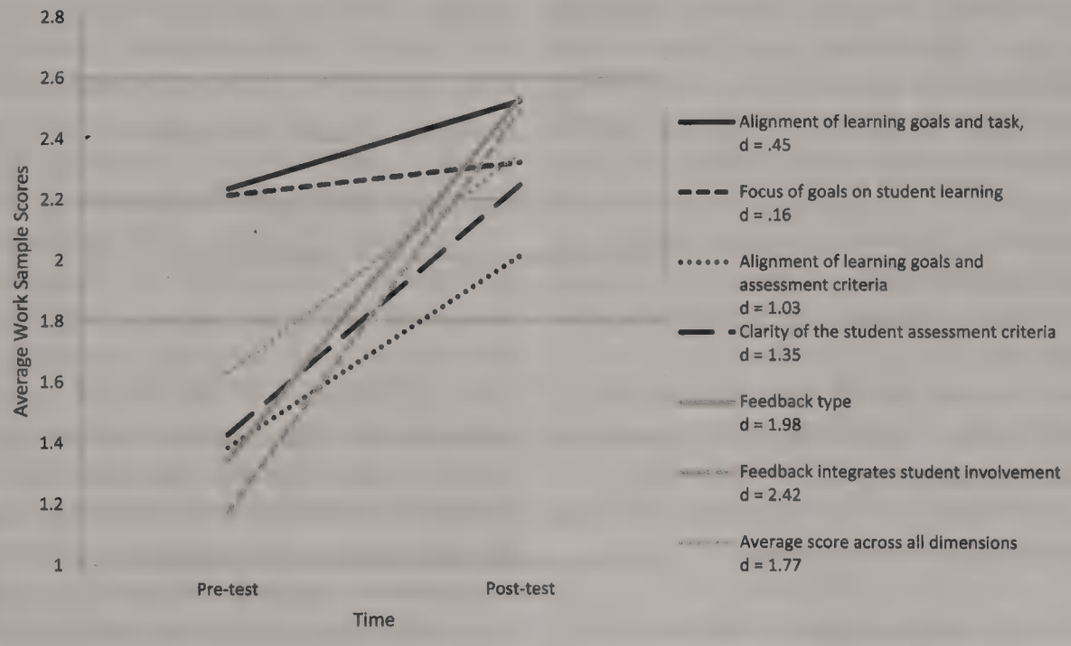

Figure 4. Average work sample scores as a function of formative assessment dimension and time (pretest to post-test; $N = 47$).

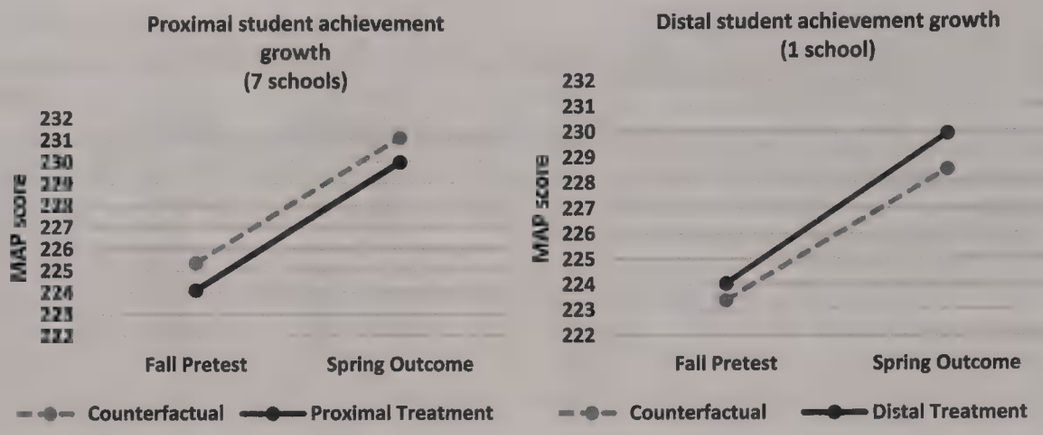

Figure 5. Adjusted means for mathematics achievement growth from fall pretest to spring outcome for treatment versus counterfactual groups.

scores ranged from −1.52 to 2.12 on the PFA assessment (mean = .41, *SD* = .80) and from −1.64 to 1.91 on the NCO assessment (mean = .31, *SD* = .87). A comparison of scores between the two subtests revealed no statistically significant difference between NCO and PFA. In order to compare scores from the AWSM field test sample to the normed LMT sample, one-sample *t*-tests were performed. Results indicate that the study sample scored significantly higher than the normed sample on both the NCO subtests, indicating that the study sample had greater pedagogical content knowledge at AWSM course entry than did the population of teachers who had previously taken the LMT. LMT scores did not significantly predict change in teacher work sample scores, indicating that prior mathematics pedagogical content knowledge was not a moderator of these effects. Therefore, AWSM improved teachers' formative assessment practice regardless of their mathematics instruction entry skills.

Student Achievement

For proximal student achievement (PD year, 7 schools), results indicate that the counterfactual group scored higher than the treatment group at the fall pretest ($p < .0001$, Glass's $\Delta = -.08$) and at the spring outcome ($p < .0001$, Glass's $\Delta = -.07$). However, the treatment x time interaction was not significant, suggesting that there was no difference in growth from fall pretest to spring post-test for students who were exposed to the AWSM PD program versus the previous year's students of the same teachers who were not exposed to AWSM.

For distal student achievement (1-year post training, 1 school), there was no significant difference between the treatment and counterfactual groups at the fall pretest or the spring outcome. In addition, the treatment x time interaction was not significant (see Figure 5).

Focus Groups

In early focus groups, teachers reported that they clearly communicated learning goals and success criteria to students, and used ungraded practice quizzes so students would understand how well they were mastering the learning goals. Teachers reported that this helped clarify their teaching and let students know what they should expect to learn. Further, having clear learning goals and success criteria facilitated communication with students and parents. Teachers said that formative assessment data helped them work with students at various levels of understanding, by changing the structure of the classroom so that students were working on different assignments based on their level of mastery of the learning goal. This helped each group focus on the next step in the learning progression.

According to teachers, AWSM helped them realize that students should be involved in the formative assessment process to increase student understanding and accountability and enable them to be more independent learners. For example, teachers gave students success criteria to use in self-assessing their level of understanding relative to the learning goal. They also paired students for more formal peer assessment. This required teachers to teach students how to provide feedback to one another and to monitor the peer feedback process. As teachers had more practice with involving students in the formative assessment process, they reported more successes.

In later focus groups, as AWSM progressed and teachers got specific suggestions in implementing formative assessment strategies, teachers at all schools began using the strategies more often and in a more efficient way. For example, teachers at all schools used more ungraded activities and found ways of providing feedback. They reported implementing low-risk ungraded quizzes as checkpoints; students working on these quizzes to get ready for the summative assessment could redo their work until they achieved mastery. In order to give feedback efficiently, one teacher arranged his/her classroom in centers to allow for more opportunities for one-on-one feedback. Another

teacher devised a system of two-letter codes that corresponded to items of feedback, such as "check to see if your answer is realistic (number sense)." This teacher explained, "You know, I'm trying to find as efficient a way as I can to make sure they're not practicing doing it wrong." One teacher commented that whenever he/she provided students with ungraded written feedback on homework assignments, students subsequently performed better on those topics in summative assessments.

At every participating school, teachers reported that their students initially resisted ungraded practice and expected a grade for every activity. Some of these teachers said that they persisted with the technique until students came to understand the advantages it has for them, such as not being assigned a grade until they had had sufficient practice. Teachers also linked ungraded practice activities to supporting the effort, focus, and perseverance mathematical practices of the Common Core, and also told students that ungraded assignments helped teachers understand how students were progressing to the learning goal.

At four schools, teachers revealed that they were doing learning targets differently than before, although these AWSM sessions had initially gotten the least positive teacher feedback. For example, one said, "I've always written my learning targets on the board, but now maybe they're very specific. I make sure the kids are aware of what we're doing today." These teachers also took advantage of success criteria templates provided in AWSM to create written rubrics for their students so they could better understand the steps to mastery. One described the process: "I worked off of that [template] for weeks, developing more success criteria and going into a simple rubric .... And having that as an electronic file that I could tweak and turn and change for each one of my classes, that was a huge deal."

Teachers at five schools reported changing their feedback practices to avoid focusing primarily on getting right answers. They described their struggling students as initially reluctant to engage in complex problem solving out of fear of getting a wrong answer; these students would avoid the task or try to get teachers to simplify it or tell them exactly what to do. Several teachers would supply answers to problems so that students would focus instead on the process and be able to tell whether their answer was correct. One said, "I try to implement, through the year, I don't care what the answer is. Show me how you get to the answer, all the steps. And it kind of puts them at ease a little bit that they...aren't afraid to be wrong anymore, and hopefully it'll help with that perseverance part."

At five schools, teachers said they were using peer assessment extensively. They described having students partner to discuss their approaches to homework problems or to go over in-class activities. One commented that this technique made it easier for students to get help when they were reluctant to seek help from the teacher. Teachers found that it was important to model and demonstrate to students how to give effective peer feedback. For example:

> Well, I used the Two Stars and a Wish [a peer feedback organization strategy], and I just put that right on the paper so they kind of knew ahead of time that they were going to get peer feedback. And the first time we did it, I kind of gave an example and showed some things that I wanted. I gave some counterexamples like, "Don't say, 'Nice handwriting,'" or something like that.

Teachers who used peer assessment sometimes found that students could better explain errors and next steps to peers. One said:

> When you walk around the room and you listen to their discussions, they're really good discussions. Sometimes they can point something out to another student that's at their level better than I can. Sometimes, we come in it from the teacher point of view, and the kid point of view they're struggling along with them and they find ways to get through to them and communicate to them that we hadn't thought about.

Another teacher said, "I've had several kids, when working together, say 'Oh, now I understand it because so and so explained it to me.' And it's like that's good because sometimes they'll say something a little bit differently, and it makes more sense." Some of the teachers pointed out that effective peer feedback helps the student giving feedback as well as the one receiving it in any interaction. For example:

> [T]heir feedback was actually better than their actual work on the original assessment .... As far as the actual, the task itself, there were some kids who did really, really well and some kids not so much, but the actual feedback, even the kids who didn't do so well on the [assignment], they started recognizing, "Oh yeah, that would have been a better approach" and so they actually...were learning even though they didn't understand how to do it themselves initially. I think by them reading someone else's they actually made sense of it.

Some teachers mentioned that peer feedback helped students understand what teachers are looking for when

they ask students to show their work—the steps in the students' approach to problems. One mentioned pairing students who show their work for peer assessment with those who generally do not, because peer feedback about how to show work was more effective at getting students to try it than was teacher encouragement.

At two schools, teachers were initially concerned that students would be unkind to one another during peer feedback. This turned out to be a problem when peer feedback was given in written, anonymous form. However, when teachers then tried structured, face-to-face peer assessment, it worked well: "I find when they verbally feedback to their peer, they're much nicer, it's more constructive, and it's actually a lot more helpful."

At three schools, teachers also focused on student self-assessment. They gave students tracking sheets of success criteria so they could understand their progress. One said, "They'll start on what they know, but then they actually take ownership of saying, 'Oh, I haven't mastered this.' And then they start testing their own learning." Another teacher described how he/she used red, yellow, and green cards that the students would periodically hold up to communicate their understanding of the learning objective. This helped him/her know when to move on and when to spend more time on a concept. Regarding self-assessment strategies, one teacher said, "it's also teaching them to be a little bit more independent workers, and be problem solvers on their own, and figuring it out." Teachers at these three schools said they looked forward to planning for student self and peer assessment from the beginning next year to establish routines and help students recognize their progress toward mathematics learning goals.

Discussion

The AWSM professional development was feasible to implement in an actual school setting. The work samples helped to ground assessment discussions in teachers' actual practice. The emphasis on formative assessment as a process including learning goals, success criteria, aligned tasks, and feedback encouraged teachers to see it as a way of framing instruction, rather than as something separate from the teaching/learning cycle. The developers made adjustments to the AWSM schedule and materials over the three iterations in order to respond to participants' performance and suggestions.

Effect sizes for changes in assessment work sample scores from pretest to post-test indicated that teachers overall demonstrated significant improvements in their formative assessment practices, with the greatest improvement in the two dimensions involving feedback and on the two dimensions involving assessment criteria. Participants showed the least improvement in the two dimensions involving learning goals, in part because of relatively high baseline scores. The pretest of mathematics pedagogical content knowledge did not moderate the pre–post difference in teachers' assessment skill. Overall, these findings suggest that the AWSM professional development program improved teachers' formative assessment practice, particularly in the areas of assessment criteria and feedback, and that it did so regardless of teachers' prior mathematics pedagogical content knowledge.

Although literature on formative assessment documents learning gains associated with higher-quality practices (e.g., Black & Wiliam, 1998a, 1998b), in this study there was no statistically significant positive impact of AWSM on student achievement as measured by MAP during the training year, despite the change in teacher practice. MAP data on the post-training-year were available only for one school, and also did not show statistically significant impact. However, because AWSM was implemented over an entire school year and because it encouraged teachers to think of formative assessment as an approach to instructional practice affecting the whole teaching-learning cycle, it is unlikely that student impacts would be seen in a training year. Future research will focus on student achievement outcomes in the year prior to AWSM professional development implementation, when teachers can plan to use the AWSM strategies from the beginning of the year.

In focus groups, participating teachers said that their students initially had the same problems reported in the literature with student motivation for learning mathematics, such as difficulty with engagement and persistence, especially with challenging problems. Their students also were reluctant to be wrong, to show work, and to do work that was ungraded. When AWSM teachers were able to involve students in understanding learning targets and in tracking progress on success criteria, they found that students were more likely to seek clarification of the goals and contribute to development of the success criteria. Teachers were able to group students to work together supportively, and to implement student peer assessment and self-assessment, even with struggling students. With the peer assessment process, using success criteria focused students on providing feedback relative to levels of performance, not just about whether the answer was right. Teachers were able to implement ungraded assignments by explaining to students their purpose, although some students struggled with this because students were used to receiving "points" as an indication of proficiency. However, when practice activities in the classroom were no

longer high-stakes and graded, students were more willing to engage and persist in more complex problem solving. Because of the importance of complex problem solving to the mathematical practices aspect of contemporary standards, and literature supporting the role of formative assessment in motivation and persistence, future research with AWSM will focus on measures of motivation in mathematics, and performance in solving complex problems (which was not addressed by the assessments we used as the achievement outcome in this study).

Teachers were able through the AWSM sessions to share their assessment practices and learn from their peers. With peer support, they were able to adjust their mindsets about what students were able to do when they shifted from a teacher-centered classroom to a more student-centered approach. Finally, teachers recognized that implementing a formative assessment process did not mean adding another initiative to an already full plate of programs but was part of effective teaching and student learning.

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[1]Only one pilot school (three teachers), which received PD in the 2012–2013 school year, provided data for the distal outcome (one year post training).

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Components of Place Value Understanding: Targeting Mathematical Difficulties When Providing Interventions

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Place value understanding requires the same activity that students use when developing fractional and algebraic reasoning, making this understanding foundational to mathematics learning. However, many students engage successfully in mathematics classrooms without having a conceptual understanding of place value, preventing them from accessing mathematics that is more sophisticated later. The purpose of this exploratory study is to investigate how upper elementary students' unit coordination related to difficulties they experience when engaging in place value tasks. Understanding place value requires that students coordinate units recursively to construct multi-digit numbers from their single-digit number understandings through forms of unit development and strategic counting. Findings suggest that students identified as low-achieving were capable of only one or two levels of unit coordination. Furthermore, these students relied on inaccurate procedures to solve problems with millennial numbers. These findings indicate that upper elementary students identified as low-achieving are not to yet able to (de)compose numbers effectively, regroup tens and hundreds when operating on numbers, and transition between millennial numbers. Implications of this study suggest that curricula designers and statewide standards should adopt nuances in unit coordination when developing tasks that promote or assess students' place value understanding.

Place value understanding requires the same activity that students use when developing fractional and algebraic reasoning (Norton & Boyce, 2015), making this understanding foundational to mathematics learning. However, many students engage successfully in mathematics classrooms without having a conceptual understanding of place value, preventing them from accessing more sophisticated later. With that said, many of these students are still not successful on place-value related items in upper elementary. Four in seven questions on the 2015 National Assessment of Education Progress (NAEP, 2015) relate to place value understandings with only 40% of fourth-grade students performing at or above the proficient level (NAEP, various years 1992–2015). Wright, Ellemor-Collins, and Tabor (2012) have investigated upper elementary students' place value development and provided detailed means of instruction and assessment for low-achieving students. However, absent from their work are detailed stages of conceptual place value understanding illustrating degrees of underlying mental activity in low-achieving students.

Battista (2012) argues that for an educator to "remediate a learning difficulty requires a more detailed picture" (p. 7), as educators are often not aware of students' underlying mental activity. In this study, we frame this mental activity with unit coordination literature (Norton & Boyce, 2015), to describe students' construction of composite units (groups of items composed of more than one item) and their ability to coordinate these units when understanding place value. The purpose of this study was to investigate elementary low-achieving students' unit coordination when engaging in place value tasks. We define a "low-achieving student" as a student achieving below standards established by research related assessments and tasks (Ginsburg & Baroody, 2003). Our inquiry focused on the following research questions:

1. What types of unit development difficulties are prevalent in the place value understanding of these low-achieving students (Grades 3, 4, and 5)?
2. How do these difficulties appear to manifest themselves as possibly limiting low-achieving students' development of place value concepts?

In this study, the authors adopt the constructivist paradigm, meaning that students learn mathematics through their active engagement with mathematics materials (Clements & Battista, 1990). We adopt this epistemological lens as informing our conceptual framework and review the literature on children's grouping and counting actions for the construction of multi-digit number understanding.

Literature Review and Conceptual Framework

Research shows that students are capable of constructing place value understandings earlier than previously thought (Resnick, 1984). Further, researchers report that children who can engage with multiple forms of activity (i.e., counting objects, grouping, estimating) construct more robust multi-digit understandings (e.g., Chan, Au, & Tang, 2014; Fuchs et al., 2010; Resnick, 1984). However, educational researchers do not yet know which of these forms of activity can best be leveraged to support low-achieving upper elementary students. In the following sections, we consider how unit coordination and conceptual place value relate to students' place value development.

Unit Coordination as a Conceptual Framework

Place value is grounded in unit coordination. Students first construct number through a building up of empirical activity into an abstract series of coordinated actions whereby part-whole understandings are reorganized and extended to construct number as a mental object to act upon (Norton & Boyce, 2015; Steffe, 1994, 2001). The construction of mental objects forms the basis for unit construction and coordination, as students rely on concrete, pictorial, fingers, symbolic numerals, and language to evidence internalized (being able to mentally re-imagine contextual actions) or interiorized (being able to draw on de-contextualized actions) actions.

When students solve problems with multi-digit numbers, they might draw upon forms of unit construction and coordination. Students understanding how proportional relationships grow by powers of 10, suggests that they would need to coordinate as many as four units (i.e., $10^{\circ} \times 10^{1} \times 10^{2} \ldots$). For example, for students to conceptually understand 1,000, they would need to understand how it relates proportionally to 100 and 10,000 ($10^{2} \times 10^{1}$ and $10^{4}/10^{1}$). This requires multiplicative operations of exponential numbers, indicative of relatively more units to coordinate than if the student were asked to simply understand 1,000 in isolation (10^{3}). Students unable to coordinate these exponential relationships might simply describe larger numbers (similar to what is illustrated above) as smaller numbers with several zeros and use language such as, "one with three zeros" or "one-thousand" but not be able to represent the value of this number concretely. However, once number is interiorized it becomes a mental object to act upon. Norton and Boyce (2015) delineate stages of unit coordination as attributable to students' number development (see Table 1). They explain that students capable of building a composite unit with a series of ones may be utilizing one level of unit coordination (Stage 1) to understand two levels of units. However, for students to operate at Stage 2, they would need to iterate a composite unit (i.e., three is repeated five times) to build a new composite unit (i.e., fifteen).

Table 1
Three Stages of Unit Coordination

	Students' Mental Actions
Stage 1	Students rely on one level of units (i.e., units of "one") and can build towards the coordination of two levels of units.
Stage 2	Students rely on two levels of units (i.e., units of "five" as composed with "five units of one") and can build toward the coordination of three levels of units.
Stage 3	Students rely on three levels of units (i.e., units of "fifteen" as composed with "five units of three").

Note. Table interpreted from Norton and Boyce's (2015) Table 1, p. 212.

Finally, Norton and Boyce (2015) explain that students at Stage 3 are able to understand simultaneously how all three levels of units relate, as they have developed mental structures for number. For example, a student at a Stage 3 level of understanding can understand how a unit of "one" relates to a composite unit of a composite unit (i.e., three sets of five). Students unable to coordinate all three levels of units simultaneously may build toward these mental structures through a "counting by" strategy or may count all. When developing Stage 3 unit coordination, students in early elementary grades are required to (1) develop mental activity necessary for conceptual place value development, and (2) develop units through counting and grouping. These two themes will be discussed more fully in this article.

Mental Activity Necessary for Conceptual Place Value Development

Procedural knowledge should be an extension of conceptual knowledge (Hiebert & Lefevre, 1986). Clark and Kamii (1996) found that first- through fifth-grade students were capable of constructing tens and ones for conceptual place value development. This finding seemed to occur when tasks afforded students opportunities to operate on multi-digit addition numbers and develop their own procedures for these operations. Thus, students were much more capable when they constructed procedures and relied on their internal mental activity.

Battista (2012) delineated a set of learning progressions that explain mental activity that students' rely on when developing conceptual place value. Seven levels delineate these actions and operations that ultimately describe students as beginning with concrete manipulative engagement, then transitioning toward abstract number engagement, and eventually developing algorithmic

procedures for multi-digit numbers (p. 10). The stance that Battista takes on his learning progressions is that educators need to transition from a deficit model toward a preventive model when assisting low-achieving students in their place value development. However, missing from Battista's delineation of these levels is how to serve low-achieving students who are operating in schools that use intervention models. In other words, how can educators intervene to support low-achieving students who are already many school years behind in meeting statewide standards and classroom objectives?

Ellemor-Collins and Wright (2011) studied conceptual place value development with low-achieving students in the upper elementary grades. Similar to Battista (2012), Ellemor-Collins and Wright found that students struggling with place value understanding should engage in tasks that extend from concrete to abstract material, and extend from place values of 200 and beyond. Furthermore, students capable of complex unit exchanges (i.e., transitioning between ones to tens to hundreds back to ones) were more capable of extending number understanding beyond 1,000.

Wright et al. (2012) delineate instruction for low-achieving students, and connect number "structures" to conceptual place value development. They define this *structuring of numbers* as students' understanding that number is a structure and structured with smaller structures (similar to part-whole reasoning). Three sets of mental activity are required for structuring of numbers: (a) combinations and pattern development, (b) part-whole number construction, and (c) relational thinking. These are very similar to unit coordination. However, these types of mental activity do not explain the degree of unit coordination students may be engaging with as tasks change.

In addition, Wright et al. (2012) posit that three dimensions should be used when supporting students' conceptual place value development. These dimensions include: (a) extending ranges of numbers, (b) making the increments and decrements of number more difficult, and (c) distancing the setting. Absent from this discussion are trajectories that explain specific degrees of change in mental activity when understanding place value. We believe that by adopting this unit coordination lens we can explain more detail regarding the degree of change in students' responses.

Structuring of numbers explains one form of mental activity students develop with conceptual place value. However, another form of mental activity that children develop with conceptual place value is a mental number line development (Dehaene, Izard, Spelke, & Pica, 2008; Moeller, Pixner, Kaufmann, & Nuerk, 2009; Siegler & Booth, 2004). Quite often mathematical tasks and assessments that use a physical number line measure changes in students' mental number lines. For instance, Siegler and Ramani (2008) found that when young children from low-socioeconomic status homes played games with linear number boards (similar to number lines) their estimation proficiency equaled that of their higher-achieving peers. Siegler and Booth (2004) also used a number line to measure first- and second-grade students' understanding of number. The findings indicated that when these students used a number line there were significant correlations between their estimating and mathematical achievement ($pr(15) = -.60$, $p < .01$ for first-grade students and $pr(17) = -.76, p < .01$ for second-grade students).

Many findings suggest that students in the early grades begin understanding magnitudes of number by relying on a mental logarithmic number line before developing a mental linear number line (Dehaene, 2011; Dehaene et al., 2008; Siegler & Booth, 2004). However, Moeller et al. (2009) found that children may be relying on two mental linear number lines (one for numbers 1–10 and one for one-digit and two-digit numbers 1–100). Moeller et al. (2009) explain why these findings conflict with Dehaene et al.'s (2008) logarithmic number line theory, as children in Moeller et al.'s study were shown Arabic numerals and children in Dehaene et al.'s study were verbally told numbers. Thus, it seems symbols and verbal number words may be processed quite differently by young children when constructing multi-digit numbers. These findings describe the comprehensive nature of children's place value development. Children developing conceptual place value need to transition from concrete to abstract experiences, transition from low numbers to high numbers, rely upon a number as a structure, and rely upon their own mental number lines. Yet, with this comprehensive perspective on place value understanding, there is still much in the literature that is unknown.

Unit Development Through Grouping and Counting

Young children develop abstract units in upper elementary grades by engaging in physical grouping actions and spatial reasoning activities (Battista, 2004; Jones, Thornton, & Putt, 1994; Long & Kamii, 2001; McGuire & Kinzie, 2013; Reynolds & Wheatley, 1996; Winer & Battista, 2015; Wright et al., 2012). For the purpose of this study, we will focus mainly on relationships between grouping and unit coordination. McGuire and Kinzie (2013) found that when young children grouped objects in units of 10, they attended to groups of tens in numbers, but could not coordinate tens and leftovers. Fuson et al. (1997) found children constructed multi-digit number understandings through a coordination of decades and ones, whereby two-digit numbers were unitized by tens (i.e.,

fifty) and by ones (i.e., three). Fuson et al. (1997) explain that children in their final stages of multi-digit number understanding need to understand numbers either by a sequence of tens and ones (i.e., ten, twenty, thirty, thirty-one, thirty-two) or by separate tens and ones (i.e., one ten, two tens, three tens, and one, two is 32).

Strategic counting is one means in which grouping and counting have been found to be effective for developing place value understanding abstractly (Chan et al., 2014; Steffe & Cobb, 1988). Chan et al. (2014) defined strategic counting as students' grouping of objects when counting. Essentially, for students to be successful in with place value, they need to build operations of numbers by first internalizing counting actions and their associated coordinated actions before interiorizing the operations as mental structures (Chan et al., 2014; Norton & Boyce, 2015; Resnick, 1984; Steffe & Cobb, 1988).

Steffe and Cobb (1988) initially proposed that students compose and coordinate units when counting. Through counting, students understand units as sets of ones prior to the construction of tens and ones. Essentially, beginning with a number-word sequence, children are able to attribute their number-word sequence to perceptual items before representing them with fingers or imagined figurative material.

Chan et al. (2014) found that first-grade students' strategic counting abilities related to their multi-digit number understanding in second grade. Additionally, young children strategically counting in different bases had a wide variety of reasoning about place value number understandings (Slovin, 2011). This suggests that children need to do more than strategic counting to understand place value (2011).

Summary

Much of the research in mathematics education provides a multi-faceted perspective on how low-achieving students conceptualize place value. However, educational researchers do not yet empirically understand relationships between low-achieving students' unit coordination and their place value development.

Much of the place value learning research within the domain of students' number understanding is in the lower elementary grades. Those who study upper elementary students' place value development describe learning progressions, where "representations of learning rely on predetermined benchmarks for student achievement across grade levels" (Weber, Walkington, & McGalliard, 2015, p. 254). The distinction here is that a learning trajectory explains "how students might engage with tasks, reflect on tasks, and develop knowledge through work on tasks" (Weber et al., 2015, p. 254). In addition, by adopting unit coordination as a theoretical model our findings can uniquely explain degrees of cognitive change with a new perspective on student learning.

Methods

This investigation was part of a larger study that used a sequential explanatory mixed-methods design to categorize student responses when understanding place value (Creswell, Plano Clark, Gutmann, & Hanson, 2003). The conceptual framework we described earlier grounded our research design, as we used student responses (qualitative data) to construct conceptual measures of student understanding (quantitative data). This article reports on the analysis of the qualitative data because nuances in student responses were analyzed with descriptive statistics. We considered the quantitative data as representing the frequency of student responses but not as an objective representation of a reality.

Participants and Setting

The participants in this study were upper elementary students referred for participation by their classroom teachers (in the school setting) or their parents (in the clinic setting) because they had experienced difficulties learning school mathematics. More specifically, teachers and parents deemed students as "low-achieving" after students failed to meet research-based standards at the same rate as their peers. For instance, students enrolled in third, fourth, or fifth grade were included in this study because research indicates that students in this age range should begin constructing rational number understandings. Researchers included three grade levels to capture the range and variance of student understandings that may occur across this age and grade span.

None of the participants were receiving special education services for mathematics. A total of 124 participants (63 male) from two western, rural school districts participated in the study. The group consisted of 41 third-grade (ages 8–9), 61 fourth-grade (ages 9–10), and 22 fifth-grade (ages 10–11) students. We did not collect socioeconomic data on individual students; however, most students (93%) attended Title I schools, suggesting a high probability that some participants were from low-socioeconomic homes.

Twenty-six classroom teachers and researchers administered diagnostic assessments with the participants in a one-to-one setting. In this study, we describe each teacher participating as a teacher-researcher, as they were fulfilling both roles as a teacher and as a researcher. The research team trained the teacher-researchers to conduct the interview assessments during a two-day training session to increase the likelihood of reliable administration of the interview procedures. During the training session, the 26 teacher-researchers learned to administer the assessment protocol

and they had multiple opportunities to practice the assessment procedures with feedback from the researchers. Teacher-researchers practiced, reflected, and discussed the logistics and decisions that would need to make when administering the assessment and categorizing students' responses. The two-day training session increased the likelihood that all members of the data collection team were consistent in the administration of the assessment.

Assessment Procedures and Analysis

The data collected for this study is a subset of the pre-intervention data collected from the administration of the Place Value Iceberg Intervention Diagnostic Assessment (IIDA) to students scheduled to participate in intervention sessions (Westenskow, Moyer-Packenham, & Child, 2017). In contrast to other place value assessments, the IIDA was designed to be used by teachers when providing Tier II place value interventions (Assessments are available at: http://targetingmathinterventions.weebly.com). The IIDA tool explicitly illustrates which underlying understandings students successfully and unsuccessfully rely on during Tier I instruction. To develop IIDA assessment questions, the place value literature was analyzed and essential components of place value learning and the types of student misconceptions and errors were identified. During the pilot assessments, interviews were video recorded and responses were analyzed to identify categorizations of responses and to ensure that the questions adequately assessed understanding. Assessments were also piloted with teacher administrators to ensure that the categories of responses were easily understood and reliably identified by teachers. Three different grade levels of assessments were developed to reflect the progressions typically used in school instruction. Although all three levels assess the same place value components, the grade-level questions varied in complexity. For example, third-grade questions were limited to numbers one thousand and less, fourth grade were limited to one million and less, and fifth grade to one billion and less. All students were tested only on their grade level and the results of further assessment to determine if the students had mastered low levels was not reported.

This study used 13 IIDA questions which integrated the foundational activity (as described by Sarama & Clements, 2009) and different dimensions (as described by Wright et al., 2012) so we could clearly see how students used previous knowledge when structuring numbers. Unique to the mathematics education field was the theoretical lens we adopted when analyzing student responses.

The research team administered the Place Value assessment to 112 of the participants in their school of residence and to 12 participants in a university clinical setting. Assessment completion typically required 30–45 minutes per student. All assessments were conducted in a one-to-one setting. For each question, the person administering the test presented a card to the student and then read aloud the question for the card. Although students were encouraged to use mental math strategies, the interviewers provided the students with paper and pencil to use if needed. Students were asked to explain their thinking. When categorization of a response was not obvious, test administrators probed the students by asking clarifying questions. If the meaning of students' verbal responses were unclear, the students were encouraged to write down their responses for further clarification. During the assessment the test administrator either recorded the category of the response or wrote a description of the response. Students' written work was collected. Assessments conducted in the clinical sessions were videotaped, but assessments conducted in the school were not. Teacher-researchers administered the assessments in the morning and met in the afternoon with the researchers to discuss student responses. For the quantitative portion of the analysis, which investigated the prevalence of unit development difficulties, we calculated the percentage of students' responses for each category. We then used these descriptive data to consider conceptually how students' responses within each component (i.e., regrouping, counting) might relate to their ability to coordinate units when understanding place value. Using qualitative methods, we employed a thematic analysis (Patton, 1990) and open and axial coding (Strauss & Corbin, 1990) to categorize students' responses for each of the question types.

Results

We identified assessment questions within six constructs of place value, which related to students' use of unit coordination with place value: Counting, Groups of Ten, Decomposing, Regrouping, Position Relationships, and Comparing. We discuss these results in the sections below.

Counting

The Counting questions assessed students' ability to count sequences and to count across transitions (see Figure 1).

Figure 1. Example of counting questions. Adapted from Shumway (2011) number sense tasks. [Color figure can be viewed at wileyonlinelibrary.com]

Table 2
Percent of Rubric Responses to Counting Questions

Grade	Concept	Incorrect		Correct	
		Sequence	Transition	Pauses	Fluent
3	Counting backward from 106 to 92	4.9	24.4	9.8	61.0
4	Counting backward from 106 to 92	10.0	23.3	1.7	65.0
5	Counting backward from 106 to 92	0	19.0	4.8	76.2
3	Counting by 10's from 570 to 710	9.8	19.5	9.8	61.0
4	Counting by 10's from 970 to 1,110	15.0	61.7	1.7	21.7
5	Counting by 10's from 570 to 710	9.5	9.5	23.8	57.2
3	Counting by 2's from 888 to 916	17.1	39.0	12.2	31.7
5	Counting by 2's from 888 to 916	28.6	23.8	14.3	33.3

Note. $N = 3–41, 4–61, 5–22$.

We first examined students' ability to sequence numbers. The results indicate that most student (more than 85%) correctly sequenced numbers when counting backwards by ones and forwards by tens (see Table 2). However, a higher number of students (17.1–28.6%) struggled when counting by twos. An explanation for this difference may be that both counting by tens and counting backwards are done in increments of one in the tens and ones positions, while counting by twos requires students to think in increments of two.

For students who correctly counted in sequence, we next examined their counting across century transitions. Counting across century transitions requires the coordination of at least two units because the number to count on from (e.g., 99, 199) needs to be reorganized with the new century (e.g., 100, 200). We identified three categories of transition responses; Incorrect; Transitions with Long Pauses (three seconds or more), and Fluent transitions. Incorrect century transition errors occurred when students stopped counting at the century, substituted a different century, or incorrectly skipped to an incorrect millennial position.

As shown in Table 2, the analysis suggested two trends: First, students tended to make more transition errors when counting by twos than when counting backward or by tens. This may be due to the need to coordinate both groups of two and the regrouping of the tens unit when making a century transition. Second, when asked to count across the one thousands position, 61.5% of fourth-grade students made a millennial transition error (see Table 2). This action required the coordination of three units as students had to count by 1 and reorganize the ten's and hundred's places. Thus, students' coordination of units may develop slowly from grade level to grade level, but the tasks also develop requiring more sophisticated coordination with higher composite units (millennial transitions) and decomposing (backward counting).

Groups of Ten

We used two questions to assess students' use of groups of ten when interpreting numbers (see Figure 1). We identified four categories of responses for question 1; No Grouping, Partial Grouping, Immediate Grouping, and Unitizing (see Figure 2 and Table 3).

Students in the No Grouping category (18.3–24.4%) counted the hearts one-by-one, suggesting that students were at Stage 1 unit coordination (i.e., not acting upon groups of ten as an object) Students in the Partial Grouping category, (0–21.7%) counted the first two columns by ones and then used groups of ten to finish counting. Students in the Immediate Grouping category (23.3–47.6%) counted one group of ten and then counted the remaining groups by tens. These two types of responses suggest that students were at Stage 1 unit coordination, but using the material effectively to build toward coordinating two sets of units. Students in the Unitizing category (28.6–36.7%) answered 42 immediately. Their explanations suggested that they anticipated and then subitized the groups of ten. This response suggests that students were at Stage 2 in their use of unit coordination.

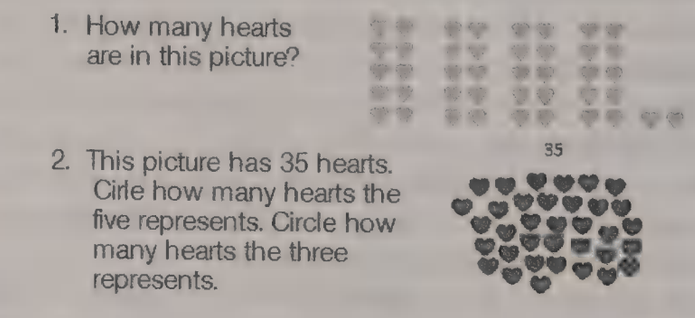
1. How many hearts are in this picture?
2. This picture has 35 hearts. Cirle how many hearts the five represents. Circle how many hearts the three represents.
35

Figure 2. Groups of ten questions. [Color figure can be viewed at wileyonlinelibrary.com]

Table 3
Percent of Students in Each Group of Ten Category

Grade	Concept	Rubric Percent			
		No Grouping	Partial Grouping	Immediate Grouping	Unitizing
3	42 hearts grouped in 10's	24.4	12.2	31.7	31.7
4	42 hearts grouped in 10's	18.3	21.7	23.3	36.7
5	42 hearts grouped in 10's	23.8	0	47.6	28.6

Note. $N = 3–41, 4–61, 5–22$.

Table 4
Percent of Rubric Responses to the 35 Hearts Question

Grade	Concept	Percent Incorrect	Percent Correct	
		No Place Value	Counts 30	Unitizing
3	Model 3 and 5 in 35	39.0	17.1	43.9
4	Model 3 and 5 in 35	32.8	26.3	41.0
5	Model 3 and 5 in 35	61.9	0	38.1

Note. $N = 3–41, 4–61, 5–22$.

The second Groups of Ten question assessed students' symbolic understanding that two digit numbers consist of groups of ten and single units. Three categories of responses were identified; No Place Value, Counts 30, and Unitizing) (see Table 4). Student responses in the No Place Value category circled five hearts and then three more hearts. (32.8–61.9%). This response did not reflect an understanding of the difference between the place value positions. This response also suggests students were not coordinating place value units, indicative of Stage 1 unit coordination. In contrast, student responses in the other two categories did reflect the place value positions as students circled five hearts and then circled 30 hearts. The difference between the two categories was that students in the Counts 30 category (0–26.3%) counted and then circled 30 hearts while students in the Unitizing category circled the remaining hearts without counting (see Table 4), The Counts 30 category is indicative of Stage 1 coordination whereas the Unitizing category is indicative of Stage 2 unit coordination.

Decomposing

To assess decomposing, we used an addition and a multiplication question (see Figure 3). The questions assessed students' use of Stage 2 unit coordination. Through these tasks students were required to act upon units of ten by partitioning composite units and then disembedding these composite units with their complements of ten. Students need to coordinate at least two units to be successful in the addition task and possibly three units to be successful in the multiplication task.

These two questions revealed the degree of difficulty many students had when decomposing numbers. For the third-, fourth-, and fifth-grade students 12.2, 18.0, and 18.2%, respectively, percent of the students did not write a correct addition combination (see Table 5). The percent of third-, fourth-, and fifth-grade students who identified three combinations were 56.1, 42.7, and 45.5%, respectively. The most common combination written was 100 + 20.

For decomposing using multiplication, the percent of third-, fourth-, and fifth-grade students who did not write a correct combination (120×1 or 60×1 was accepted) were 22.0, 14.8, and 36.4%, respectively, and the number of students who wrote four combinations was 4.9, 9.8, and 0%, respectively. An important difference between students' accuracy when decomposing additively versus multiplicatively was students often additively compensated with a "+ 1" or a "+ 2" strategy rather than using groups of 10. The need to use multiplicative decomposing prevented students from using this compensation strategy when decomposing multiplicatively. Further, students responding in this way were at a Stage 1 or 2 unit coordination because they could coordinate one or two

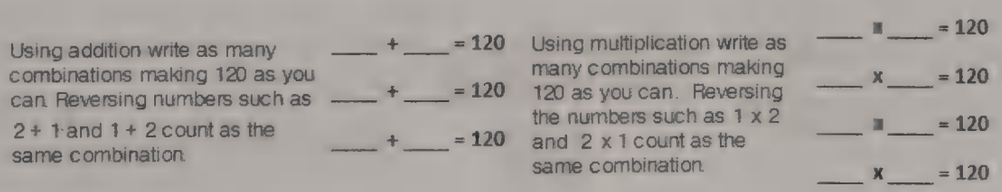
Using addition write as many combinations making 120 as you can. Reversing numbers such as 2 + 1 and 1 + 2 count as the same combination.
___ + ___ = 120
___ + ___ = 120
___ + ___ = 120
Using multiplication write as many combinations making 120 as you can. Reversing the numbers such as 1 x 2 and 2 x 1 count as the same combination.
___ x ___ = 120
___ x ___ = 120
___ x ___ = 120
___ x ___ = 120

Figure 3. Examples of the addition and multiplication decomposing questions.

Table 5
Percent of the Number of Responses for Each Composing/Decomposing Question

Grade	Question	Number of Combinations 0	1	2	3
3	Addition combinations to 120	12.2	29.3	2.4	56.1
4	Addition combinations to 120	18.0	24.6	14.8	42.7
5	Addition combinations to 120	18.2	22.7	13.6	45.5

Grade	Question	Number of Combinations 0	1	2	3	4
3	Multiplication combinations to 60	22.0	24.4	41.5	7.3	4.9
4	Multiplication combinations to 120	14.8	26.2	26.2	23.0	9.8
5	Multiplication combinations to 120	36.4	50.0	13.6	0	0

Note. $N = 3–41, 4–61, 5–22$.

units, but struggled to multiplicatively understand the operative structure of unit needed for multiplication.

Regrouping

Regrouping requires students to act upon their counting and place value construction to regroup numbers. The three skills relevant to unit coordination assessed in this component were addition regrouping, subtraction regrouping, and adding/subtracting one at the transitions to tens and thousands (see Figure 4).

For the regrouping questions we identified three types of regrouping responses: No Regrouping, Incorrect Procedure, and Correct Regrouping (see Table 6). In the first level, No Regrouping, students did not attempt to regroup. For example, to name 2 hundreds, 13 tens, and 1 ones, some students wrote 2,131 and when subtracting, some students subtracted the digit of the least value from the digit of the greatest value. These responses suggest that students were at Stage 1 unit coordination, as they did not appear to understand the conceptual need for a reorganization of units when operating on multi-digit numbers.

In the Incorrect Procedure category, students attempted to regroup but made procedural errors, such as adding 10 instead of 1 to the next position when adding or not exchanging ten rods for one hundred (fifth grade) or ten blocks for one ten rod (third, fourth, and fifth grades). These responses suggest that students were at Stage 1 unit coordination because students had procedures not grounded in conceptual understandings of number. In the Correct category, 85.4% of third graders correctly regrouped ones in a two-digit number, and 39.3% of fourth graders and 54.5% of fifth graders correctly regrouped tens in a three-digit number when naming the modeled numbers (see Table 6). Less than one-half of the students correctly modeled regrouping when subtracting. These responses suggest Stage 2 or 3 unit coordination, as students were either building three levels of units or utilizing three levels of units to coordinate trading and (de)composition of number.

The Before and After regrouping questions assessed students' ability to regroup when crossing transitions (see Figure 5). These types of questions required students to engage in Stage 2 or 3 unit coordination.

Analysis of the Before and After Question results yielded four categories of responses; Last Digit, Both Incorrect, One Correct, and Both Correct (see Table 7). For the Last Digit category, students added and subtracted one to and one from the last non-zero digit in the number. For example, students would respond that 102,000 and 104,000 were one less and one more than 103,000. This suggests that students were not coordinating units of one thousand, but were focused on counting in sequence without consideration of the place value unit of the non-zero digit.

In the second category, students incorrectly attempted regrouping. For example, some student responses to one

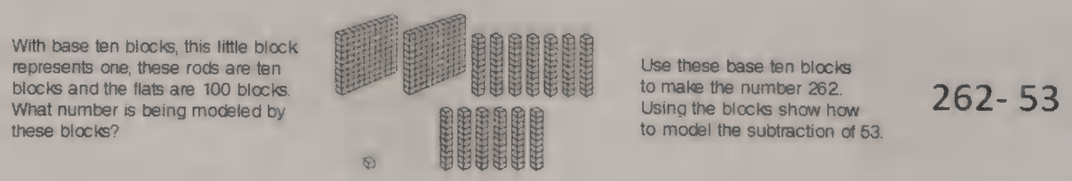
With base ten blocks, this little block represents one, these rods are ten blocks and the flats are 100 blocks. What number is being modeled by these blocks?
Use these base ten blocks to make the number 262. Using the blocks show how to model the subtraction of 53.
262- 53

Figure 4. Example of an addition and a subtraction regrouping question.

Table 6
Percent of the Number of Responses for Each No Regrouping/Incorrect Procedure/Correct Procedure Question

Grade	Concept	No Regrouping	Incorrect Procedure	Correct
3	Regroup 5 tens 12 ones	7.3	7.3	85.4
4	Regroup 2 hundreds 13 tens 1 one	29.5	31.1	39.3
5	Regroup 2 hundreds 13 tens 1 one	18.2	27.3	54.5
3	81–3	4.9	46.3	48.8
4	262–53	3.3	52.5	44.3
5	632–253	9.1	68.1	27.7

Note. $N = 3$–41, 4–61, 5–22.

less than 103,000 were 103,999; 103,009; or 193,999. In the third category, One Correct, students answered one question correctly which typically was the "one after question," suggesting that students could operate at Stage 1 or 2 unit coordination. Students would need to have an operative structure for addition and subtraction relationships to be successful with the "one less" question, indicative of level three unit coordination. In the Both Correct level, there was a large contrast in the number of third-grade students (41.5%) and the number of fourth (12.3%) and fifth (14.3%) grade students who answered both before and after correctly (see Table 7). These findings suggest that students may have operative procedures for lower digit numbers, but struggle with recursive operative relationships when extending these procedures to higher digit numbers.

Relationships

Relationship questions assessed students' conceptual understanding that each place value position is ten times the position to the right and 1/10 of the position to the left (a Stage 3 unit coordination requirement). Three types of questions were used to evaluate students' understanding; (a) How Many Times Larger; (b) Multiplying by Multiples of Ten; and (c) Dividing by Multiples of Ten. At the beginning of the relationship component of the assessment, test administrators asked students how many times larger 200 is than 20. In response, 34.1% of third graders, 37.7% of fourth graders and 22.7% of fifth graders responded with 10 times larger. The other two groups of Relationship questions asked students' to mentally calculate the solution to four multiple of ten multiplication and four multiple of ten division expressions of increasing complexity (see Figure 6).

If you were counting what number would come one before 203,000. What number would come one after 203,000

203,000

Figure 5. Example of before and after question.

Table 7
Percent of Rubric Responses to Before and After Regrouping Question

Grade	Question	Last Digit	Both Incorrect	One Correct	Both Correct
3	500	24.4	17.1	17.1	41.5
3	770	24.4	7.3	7.3	61.0
4	203,000	49.2	13.1	16.4	21.3
5	103,000	28.6	9.5	61.9	0
5	1,030,000	19.0	33.3	33.3	14.3

Note. $N = 3$–41, 4–61, 5–22.

Most students (90% or higher) correctly multiplied at least one of the four multiplying questions (see Table 8). Typically students correctly answered 3×10, a fact which is often memorized as part of multiplication fact practice. However, less than half of the students were able to correctly solve all four multiplication/division expressions. When asked to explain their strategies, many students did not refer to place value relationships but only to adding or subtracting zeros. It is probable that students were following procedural rules and were not engaging in unit coordination. This lack of unit coordination is further supported by the low number of students who responded that 200 was 10 times greater than 20. Students did not appear to understand or use the concept that a place value position is 1/10 of the position value to the left and 10 times the position value to the right.

Most students tended to correctly solve more multiplication than division expressions. This suggests that some students struggled with the inverse relationship between multiplication and division and is indicative of students operating at Stage 2 unit coordination.

Comparing Numbers

The Comparing Numbers question relevant to unit coordination assessed students' understanding of number density by asking students to identify a number between two given numbers (see Figure 7). Correctly naming numbers between two or three digit numbers required students to coordinate two units. Correctly naming numbers between numbers in the thousands and millions required students to coordinate three units. Students needed to use the relationships established when coordinating two units to understand the abstract relationships when coordinating three units.

This analysis revealed four categories of responses: Correct; No number; Last Non-zero Digit; and Others. For smaller numbers such as between 30 and 40, and between 600 and 700, 73.2 and 70.6%, respectively, of the third-

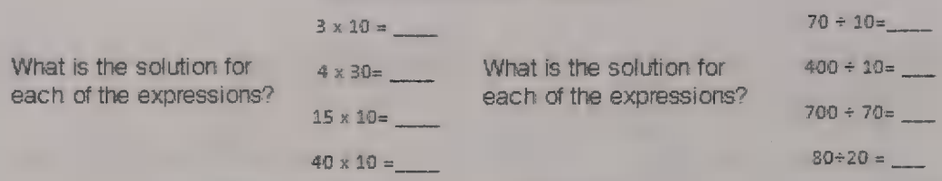
What is the solution for each of the expressions?
3 x 10 = ____
4 x 30= ____
15 x 10= ____
40 x 10 =____
What is the solution for each of the expressions?
70 ÷ 10=____
400 ÷ 10= ___
700 ÷ 70= ___
80÷20 = ___

Figure 6. Examples of multiplying by multiples of ten and dividing by multiples of ten questions.

grade students correctly identified a number (see Table 9). However, for larger numbers in the thousands and millions periods, less students correctly identified numbers between the two given numbers. This change in accuracy may be due to students' inability to operate at Stage 3 unit coordination. Operating with larger and less familiar numbers requires more abstract, recursive, unit structure understandings.

Students who gave No Number category responses typically first hesitated and then stated there were no numbers between the two given numbers. Students who gave Last Non-zero Digit category responses added or subtracted one to the last non-zero digit of one of the numbers (e.g., 412,000 is between 410,000 and 411,000). These two types of responses suggest that students were relying on sequence counting and were not coordinating the place value units. Students who gave Other category responses named numbers that were outside the range of the two numbers given and were not of the Last Non-zero Digit category (e.g., 411,001 or 499,000 for numbers between 410,000 and 411,000).

Discussion

In this paper, we examined elementary students' unit coordination when engaging in tasks to explain Place Value understandings. In this section, we discuss the difficulties students exhibited with place value concepts and the educational implications of our findings.

Difficulties Children Experienced When Understanding Place Value Concepts

The third-, fourth-, and fifth-grade students in this study struggled with foundational aspects of place value relationships within multi-digit numbers. This was associated with their inability to flexibly operate on numbers and use their knowledge of place value between millennial numbers. Wright et al. (2012) posit that students need to engage with hundreds materials when extending their understanding to 1000. However, in our tasks we only gave the students in this particular study symbolic material; this prevented them from be able to make connections between concrete and symbolic material when engaging with millennials. Students were also unable to treat digits in their respective place value positions as anything more than the value of the digits themselves when regrouping and unitizing (i.e., the "3" in 34 is treated as a 3, not a 30). These findings are not new as they are very similar to the findings from Fuson et al. (1997) wherein students struggled when transitioning from single-unitary digit numbers to multi-digit numbers. These findings suggest that students in this study may have been low-achieving because they were not yet able to interiorize concepts of ten (Fuson et al. 1997). In addition, we posit that students were unable to coordinate three levels of units and may have benefitted from more opportunities to use concrete material to build toward three levels. The Fuson et al. (1997) task may explain when students are (un)able to coordinate two levels of units because they are not given concrete material to build toward two levels of unit coordination. For instance, because of the lack of concrete experiences, students may not be able to develop and use composite units as mathematical objects when operating on smaller and larger numbers, relatively. This aligns with Norton and Boyce's (2015) discussion on unit coordination, which

Table 8
Percent of Students Who Correctly Answered None, One, Two, Three, and Four of the Multiplying/Dividing Questions

Grade	Concept	Rubric Percent				
		None	One	Two	Three	Four
3	310, 4 × 100, 12 × 100 and 60 × 100	7.3	2.4	34.1	41.5	14.6
3	30/3, 40/10, 420/10, 400/100	17.5	17.5	32.5	20.0	12.5
4	3 × 10, 4 × 30, 15 × 10, and 40 × 10	1.6	4.9	31.1	18.0	44.3
4	70/10, 400/10, 700/70, 80/20	0	23.1	10.3	25.6	41.0
5	3 × 10, 4 × 1,000, 23 × 10 and 40 × 100	4.5	0.0	22.7	31.8	40.9
5	100/10, 6,000/10, 1200/40, 25/100	36.4	13.6	36.4	13.6	0

Note. $N = 3–41, 4–61, 5–22$.

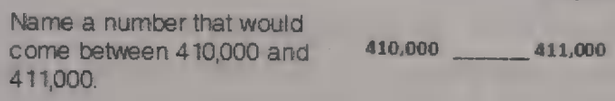

Figure 7. Example of a density question.

suggests that students at the Stage 1 unit coordination understanding are not capable of developing multi-digit or composite units absent from concrete material and that this development only allows them access to tasks that require two levels of unit coordination, not three levels of unit coordination.

Our findings also suggest that students must understand the relationships between column values (density). What this means for students is that they should understand how one place value column is one multiple of ten difference from a subsequent place value column. For students to understand and operate on multi-digit numbers, their understanding of the operational structures for multi-digit numbers must be recursive. Essentially, students should be able to extend their number understanding to larger numbers by incorporating operational structures of numbers developed through their prior unit coordination (Norton & Boyce, 2015; Wright et al., 2012). Students in this study simply manipulated the place values by adding or taking away zeros without understanding the changes in value that these actions produced. Wright et al. (2012) posit that this type of activity is what students and teachers expect when engaging with conventional place value instead of conceptual place value. We further argue that it is far more effective to understand how this conventional activity illustrates stages in students' unit coordination. For instance, when engaging in this task students were typically operating at a Stage 1 unit coordination, preventing them from accessing Stage 3. If students were able to operate at a Stage 2 they did not always have strategies (i.e., counting by multiples of ten) to build toward Stage 3. Students' inability to have an interiorized structure of the proportional relationships may explain why they had difficulty transitioning from two- and three-digit number understanding and operations to six- or seven-digit number understanding and operations.

Counting, Grouping, and Unit Coordination

We analyzed the data through the theoretical lens of students' unit coordination abilities (Norton & Boyce, 2015). Students were successful when engaging in tasks when characteristics of the tasks only required students to coordinate units at Stage 1 or 2. This was evident when students used groups to build toward three tens or "3" in 35, but they were not able to immediately draw on different groups of tens when (de)composing number. Also, students' ability to find numbers in between two multi-digit numbers seemed to require two or three levels of interiorized units.

This may explain why students in third grade were able to utilize a strategy when finding numbers in between two and three-digit numbers, but almost half of the students in fourth and fifth grade struggled to name a number in between six and nine digits. These findings suggest that students may be internalizing units to coordinate each time, but are not interiorizing recursive relationships of multi-digit number understandings as a mental structure. Wright et al. (2012) suggest that when students are developing place value they need to engage in the making of complex sets of increments and decrements with multiples of ones, tens, and hundreds. These actions further press students' development of larger numbers later. In this study, we could argue that in engaging in these increments and decrements, students are constructing operations on and around numbers, which promotes more sophisticated unit coordination.

It was also evident that, for students to provide multiple combinations for number and to develop operations for those combinations, students would need to coordinate more groups and develop more flexible, sophisticated part-whole understandings. For instance, when students were asked to describe as many as four multiplicative combinations for a number, they were required to coordinate units at stages 2 and 3, but when students were asked to simply solve multiplication problems, many of

Table 9
Percent of Rubric Responses to Density Questions

		Incorrect			
Grade	Question	No Number	Last Non-Zero Digit	Other	Correct
3	Between 30 and 40	7.3	17.1	2.4	73.2
3	Between 600 and 700	12.2	14.6	2.4	70.6
4	Between 410,000 and 411,000	48.3	1.6	9.8	44.3
5	Between 610,000,000 and 611,000,000	40.9	13.6	0	68.2

Note. $N = 3–41, 4–61, 5–22$.

their responses required them to coordinate units only at Stage 2. This finding aligns with that of Wright et al. (2012), who reported that the more number structures children create, the more sophisticated their understanding for that number.

Finally, findings from the regrouping and grouping while counting tasks suggest that the degree of the number of units needed to coordinate and the material provided to the students may (dis)allow them opportunities to build from one unit coordination stage to the next. For instance, in the regrouping tasks more than 85% of the third-grade students were successful when using a picture and two numbers to regroup. However, when the fourth- and fifth-grade students were given the same task, but with three numbers to regroup, only about half of the students were correct. Also, when third and fifth graders were asked to count by "two's" their accuracy rates were much lower than their fourth-grade student counterparts who were asked to count by "10's." These findings suggest that unit coordination became quite difficult for three reasons: (a) more numbers to coordinate required more abstract unit coordination, (b) novel units to coordinate required more flexible unit coordination, and (c) images of blocks disallowed active engagement for some students when building from one stage of unit coordination to another stage.

These findings suggest that when students in third grade were successful it was because they were given tasks that only required them to engage in Stage 2 unit coordination. Whereas fourth- and fifth-grade students were less successful when given relatively more sophisticated numbers, more units, and fewer pictorial items. These changes in assessment questions were fundamental in how students were able to access their counting, grouping, and unit coordination and suggest that students at different grade levels require different stages of unit coordination when engaging in place value tasks.

Educational Implications

These results have implications regarding the design of assessments and interventions for students' understanding of place value. For instance, it is still not clear how educators can vary their instruction to address specific underlying mental actions that low-achieving students may require who are already many school years behind in meeting statewide standards and classroom objectives. Findings from this study do not directly address these concerns, but suggest that curricula and statewide content standards do not utilize important mental activity (e.g., Common Core State Standards for Mathematical Practices, 2009; Principles to Action, 2014) necessary for low-achieving students when designing interventions. The Common Core Mathematical practices that illustrate effective student engagement is not nuanced enough to describe how student engagement may appear when students are performing well below grade level and, more specifically, how this activity should differ for conceptual place value instruction. Thus, addressing these specific concerns would benefit these students with this particular form of number development. Assessment and intervention should focus on inferred mental actions that students rely on when solving problems and not just accurate procedures. Quite often statewide standards guide students' place value assessments rather than engaging students in the necessary mental activity needed for success for these problems. Curricula designers and those designing statewide standards should consider these necessary mental actions when designing assessments and interventions. Many U.S. students without conceptual place value understandings are unable to reach Stage 3 unit coordination to successfully meet place value standards. This suggests that common standards and assessments need to change to reflect mental activity necessary for students' conceptual place value understandings.

More specifically, Wright et al. (2012) address the "structuring" of numbers students need to engage in when developing early number and conceptual place value understandings. Many of the interventions Wright et al. posit for place value development address activity with materials (e.g., incrementing and decrementing) and the proximity students have with materials (e.g., concrete, visualized concrete, verbal). Findings from this study echo these intervention models and explain the stage of unit coordination, which may be used by students when relying on activity and materials. Thus, we argue that curriculum should take on these aspects of of activity and materials while addressing the stages of unit coordination.

A final educational implication is that curricula designers should design intervention procedures that provide more nuanced options that closely align with student's place value understandings and abilities. Low-achieving students, are not yet able to master particular conceptual place value knowledge in lower grade levels, which then further distances them from achieving at the same rate as their peers in number and operations in upper grade levels. Thus, future research should use unit coordination to more fully develop learning trajectories that can support this curricula design. This new theoretical perspective can provide opportunities for educational stakeholders to consider the degrees of learning necessary for such a multi-faceted concept in mathematics.

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Instructional Supports for Representational Fluency in Solving Linear Equations with Computer Algebra Systems and Paper-and-Pencil

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This research addresses the issue of how to support students' representational fluency—the ability to create, move within, translate across, and derive meaning from external representations of mathematical ideas. The context of solving linear equations in a combined computer algebra system (CAS) and paper-and-pencil classroom environment is targeted as a rich and pressing context to study this issue. We report results of a collaborative teaching experiment in which we designed for and tested a functions approach to solving equations with ninth-grade algebra students, and link to results of semi-structured interviews with students before and after the experiment. Results of analyzing the five-week experiment include instructional supports for students' representational fluency in solving linear equations: (a) sequencing the use of graphs, tables, and CAS feedback prior to formal symbolic transpositions, (b) connecting solutions to equations across representations, and (c) encouraging understanding of equations as equivalence relations that are sometimes, always, or never true. While some students' change in sophistication of representational fluency helps substantiate the productive nature of these supports, other students' persistent struggles raise questions of how to address the diverse needs of learners in complex learning environments involving multiple tool-based representations.

Developing students' representational fluency—the ability to create, move within, translate across, and derive meaning from external representations of mathematical ideas (cf. Zbiek, Heid, Blume, & Dick, 2007)—has remained a prevalent value in school mathematics for over two decades. Representational fluency is the linchpin in the National Research Council's (NRC) (2001) description of conceptual understanding, and is increasingly important in technology rich school algebra (Heid & Blume, 2008). Indeed, representing and learning are linked as mutually supportive processes (e.g., see Selling, 2015). The purpose of this research is to explore links between instructional supports and students' representational fluency in solving problems with computer algebra systems (CASs)/paper-and-pencil.

Representational Fluency in Solving Equations

Within the landscape of school algebra reform, both the content of a functions approach to solving equations, and the mathematical practice of reasoning fluently with multiple representations in technology rich settings are clearly valued (Arbaugh et al., 2010). The context of a functions approach to solving equations with CASs and paper-and-pencil is a rich context for exploring students' representational fluency and supports for that development with secondary algebra students (Kieran & Yerushalmy, 2004), especially when CASs are treated as a representational toolkit to create and move between representations (cf. Dick & Edwards, 2008). By a functions approach to solving linear equations, we mean that "a linear equation in one variable is, in essence, a specific instance of a corresponding linear relationship (or equation in two variables)" (Cai, Nie, & Moyer, 2010, p. 178) and includes comparing relationships among graphic, numeric, and symbolic representations of functions created with technology (Cai, Nie, & Moyer, 2010, p. 178).

In research with graphing calculators, Kieran and Sfard (1999) investigate supports for students' meaningful manipulation of equivalent expressions and Yerushalmy and Gilead (1997) investigate supports for students' meaningful equation solving. Both studies suggest a teaching sequence by which graphical and numeric function table representations are introduced prior to formal symbolic representations. Yerushalmy and Gilead contend that once solving with graphs and tables, the students realized the need for more formal, exact methods of solving. Across these studies the authors report evidence of students' meaningful manipulation of symbols, supporting the need to test such instructional supports with CAS.

In contemporary research involving CAS, effects of technology-intensive instruction that emphasize a functions approach to solving linear equations lead to somewhat mixed findings on whether and how a functions approach supports students' representational fluency in solving equations. In research by Hong and colleagues (Hong &

Thomas, 2002; Hong, Thomas, & Kwon, 2000) they found mixed effects of their instructional approach that showed students how to solve equations from an algebraic, graphical, and tabular perspective with the aid of the CAS. No statistically significant gains were made on measures of routine problems, yet gains on some problems involving solving across representation types were found. Other research has made progress in characterizing students' representational fluency and problem solving (e.g., Adu-Gyamfi, Bossé, & Chandler, 2015; Adu-Gyamfi, Bossé, & Stiff, 2012), yet relatively few studies link representational fluency and instruction (see Selling, 2015 as a notable exception).

To build on this work, there is a need for greater attention to clarify how functions approaches are leveraged in CAS/paper-and-pencil environments, including how representations are sequenced and connected, and how those supports are linked to students' change in representational fluency. In a CAS/paper-and-pencil environment, we conceive of instructional supports to include a sequence of tasks, techniques, and theory that inform the teacher's instructional moves and the opportunities for students' learning experiences. This research addresses the question: How is instruction in a CAS/paper-and-pencil environment supportive of students' change in representational fluency in solving linear equations?

Conceptual Framework

We employed a learning progressions approach to elaborate both means of support for student learning and the nature of students' learning vis-à-vis change in students' representational fluency. Shown in Figure 1, a curricular framework informs the curricular progression and assessment of learning; the instructional sequence is informed by the curricular progression. Likewise, characterizations of student thinking are conceived of with respect to the influence of the instructional sequence and the manner in which learning is assessed. Our learning progression approach builds from an emergent perspective (Cobb & Yackel, 1996) in which means of support for students' learning (i.e., curriculum, instruction including classroom interaction and discourse) inform and are informed by individual learning processes. In this article, we seek to provide support for the ways in which the instructional sequence influenced students' representational fluency. This focus is shown as a dashed arrow in Figure 1.

Means of Support

Following Fonger, Stephens, Blanton, Isler, and Knuth (in press), a curricular framework specifies the targets of learning and entails big ideas, algebraic thinking practices, core

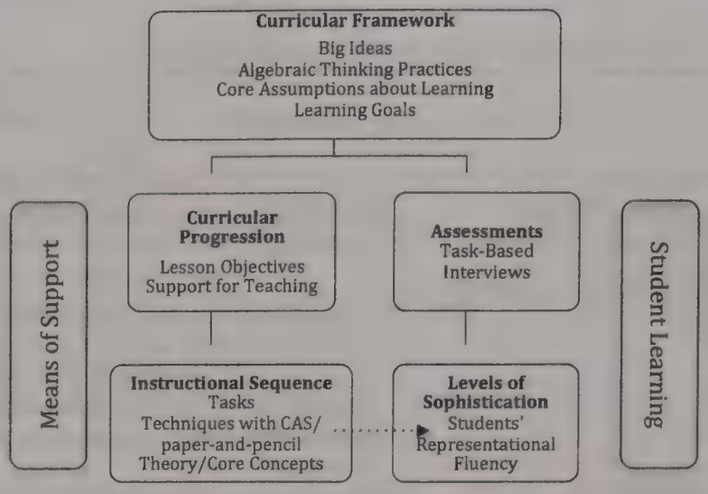

Figure 1. A learning progressions approach to investigating relations between means of support and student learning.

assumptions about learning, and learning goals. This research is rooted in two big ideas of algebra: (a) equivalence, expressions, equations, and (b) a functions approach with multiple representations. The learning goal that guided the instructional design relevant to this report is to develop representational fluency in solving problems involving linear equations in one and/or two variables. Our assumptions about learning were rooted in an emergent perspective in which interactions and discourse at the classroom level inform and are informed by students' activity and understandings. Specifically: (a) a meaningful understanding of equation solving is possible when explored through multiple technology-based representations (e.g., Kieran & Sfard, 1999), and (b) a relational understanding of the equal sign is significant for students' abilities to solve equations (e.g., Knuth, Stephens, McNeil, & Alibali, 2006).

The curricular progression includes a sequence of lesson objectives that address mathematical content, role of representations, and tool use. Support for teaching includes summaries of empirical research to inform the design and implementation of tasks with students. The instructional sequence is a string of tasks specifying "what" mathematics students are to accomplish, the techniques or "how to" complete these tasks with CAS/paper-and-pencil, and the domain-specific theory or core concepts that undergird the "mathematical understandings."

Characterizing Sophistication in Representational Fluency

A representation is something that stands for something else from some one's point of view. In solving equations from a functions approach, students may employ strategies within and across symbolic, graphic, and numeric representation types (including CAS/paper-and-pencil function tables and numeric equations such as $0 = 0$).

Table 1
A Taxonomy for Sophistication in Representational Fluency

	Sophistication	
	Lesser	Greater
Nature of Students' Discursive Activity	Student expresses an incorrect or incomplete creation, interpretation, or meaning.	Student expresses correct and complete creations, interpretations, and meanings.
Representation Types	SOLO Level	
One	*Prestructural.* Student demonstrates some procedural work within a given representation type but no understanding is conveyed. Student is not able to create and/or interpret representations of a given type with success to complete a task.	*Unistructural.* Student creates and/or interprets representations within a single type to successfully complete a task but demonstrates no or limited understanding of connections to other representation types.
Multiple	*Multistructural.* Student moves between more than one representation type to make progress toward or successfully complete a task but demonstrates no or incomplete understanding of how these representation types are related or what they mean to solve the problem.	*Relational.* Student relates at least two different representation types together with a more sophisticated understanding of how they are related, what they mean, and/or of their significance to the whole. In connecting multiple representations, a student gives a correct interpretation of an invariant feature across multiple representations or types.

Additionally, a unique feature of CAS technology is the result of evaluating an equation at values; CAS feedback yields the word "true" when an equation is evaluated at a solution to the equation, and "false" otherwise. The use of such CAS feedback to inform student learning of equivalence has been productive in previous research, and identified as an area for further investigation (e.g., Kieran & Drijvers, 2006). With an intention to build on the aforementioned studies by Kieran and colleagues, and following Kaput (1992) on the importance of studying students' representational practices with technology, in this study we treat the CAS feedback of "true" or "false" as an important representation type.

We characterize sophistication in students' representational fluency based on the number of representation types, the correctness of students' creation and/or interpretation of these representations in solving problems, and the meaning students making of their strategies. The structure of observed learning outcome (SOLO) taxonomy (Biggs & Collis, 1982) was adapted for this study to categorize levels of sophistication in representational fluency. In this case, a single assessed component is students' use of a representation type to solve a problem. The coding taxonomy is given in Table 1; greater sophistication is discerned from left to right.

In brief, when a student's strategy focuses on a single representation type to solve a task, with little or no understanding of connections to other representation types, it may be an incorrect creation and interpretation—prestructural level. A correct creation and interpretation within a representation type is unistructural. When a student's strategy involves more than one representation type, it could yield progress yet not successfully solving the task—multistructural level. Finally, a student's correct solution of a task with multiple representations with a more sophisticated understanding of how they are related, what they mean, and/or of their significance to the whole is relational. Students make a relational connection when they give a correct interpretation of an invariant feature across multiple representations or types. A typical example of a relational connection with CAS/paper-and-pencil is to solve $7x + 3 = 2x - 7$ for x by creating a graph of $f(x) = 7x + 3$ and $g(x) = 2x - 7$ and interpret the meaning of the x-value of the intersection ($x = -2$) as the solution to $7x + 3 = 2x - 7$. Another example of a relational connection is to type "$7x + 3 = 2x - 7|x = -2$" into CAS to yield "true" and interpret the meaning of this movement from a symbolic equation to CAS feedback as signifying the solution to the equation is $x = -2$. Overall, change in greater sophistication in representational fluency is interpreted as a shift in discursive activity from incorrect to correct creation, interpretation, and meaning making within and across representation types. The meaning a student draws from their creation and/or interpretation of external representations is key at both unistructural and relational levels.

Methods

To understand both how to support students' representational fluency in a CAS/paper-and-pencil

Table 2
An Overview of the Instructional Sequence

Sequence	Topic and Representation Types	Tools*
Section 1	Equivalence of Expressions	
Activity 1	Justify equivalent expressions as the same pattern, graph, table	CAS/P&P
Activity 2	Justify equivalent expressions by common symbolic form	CAS/P&P
Section 2	Transition from Expressions to Equations	
Activity 1	Explain symbolic identity equations as numerically "always true"	CAS/P&P
Activity 2	Classify equations as sometimes (intersect), always (same line), or never true (parallel)	CAS/P&P
Section 3	Solving Linear Equations	
Activity 1	Solve equations $ax + b = c$ OR $a(x + b) = c$ for x with one solution	
Part I	Compare graphs/tables $f(x) = ax + b$ or $f(x) = a(x + b)$ and $g(x) = c$	CAS/P&P
Part II	Define and verify solutions as "true" values	CAS
Part III	Create equivalent symbolic equations	P&P
Part IV	Connect a solution across graphs, tables, symbols, words	CAS/P&P
Activity 2	Solve equations $ax + b = cx + d$ or $ax + bx + c = dx + e$ for x (one solution)	
Part I	Create a series of symbolic equivalent equations	CAS
Part II	Create a series of equivalent equations, verify the solution	CAS/P&P
Activity 3	Solve equations $ax + b = cx + d$ for x with one solution	
Part I	Compare graphs/tables of $f(x) = ax + b$ and $g(x) = cx + d$ to solve	CAS
Part II	Create a series of equivalent symbolic equations	P&P
Part III	Connect "true" value across graphs, tables, symbols	CAS/P&P
Activity 4	Solve equations $ax + b = cx + d$ for x with infinite/no solutions	
Part I	Compare graphs/tables of $f(x) = ax + b$ and $g(x) = cx + d$ to solve	CAS
Part II	Create a series of equivalent symbolic equations	P&P
Part III	Connect "true" value(s) (if any) across graphs, tables, symbols	CAS/P&P

*Tools include computer algebra systems (CASs) and/or paper-and-pencil (P&P).

environment, and the nature of learning in the context in which it was supported, we conducted one cycle of a collaborative teaching experiment (CTE) (Cobb, 2000), a type of design experiment (Gravemeijer & Cobb, 2006). This methodological choice was rooted in our intent to advance an understanding of how instructional supports influenced students' representational practices. Student interviews were conducted before and after the teaching experiment.

Collaborative Teaching Experiment

We conducted a five-week CTE in three phases: design; experimentation and ongoing analyses; and retrospective analyses. The CTE design was chosen to: (a) address a problem of practice relevant to classroom teaching; (b) enhance the ecological validity of the findings per the research setting of an intact classroom; and (c) provide potentially meaningful supports to bolster opportunities for students to learn with CAS.

Participants. All student participants ($n = 31$) were enrolled in the same ninth-grade algebra course in an urban public high school in the Midwest of the United States.[1] Ms. R, a fifth-year high school mathematics teacher taught the instructional unit during the last period of the school day. Three students from this class volunteered to also participate in task-based interviews before and after the instructional unit, and have their daily classwork recorded. We selected three students, Annie, Bryon, and Carlos,[2] who demonstrated a range of mathematical abilities (mid, high, and low percent scores, respectively) relative to a class-wide pre-assessment.

Instructional sequence. The instructional design was guided by a consideration of how core concepts, techniques, and tasks encouraging using multiple representation types and tools could be coordinated and sequenced to support student learning. Table 2 outlines the content, representation types, and tools used during the instructional sequence. The focus of this study, Section 3 Solving Linear Equations, was introduced by building on students' experiences in comparing and justifying expressions related by an equal sign to identifying solutions to equations with one solution such as $ax + b = c$ and $a(x + b) = c$, or $ax + b = cx + d$ with functions $f(x) = ax + b$ or $f(x) = a(x + b)$ and $g(x) = c$ or $g(x) = cx + d$, respectively. Tasks involved creating and comparing CAS graphs and tables of equations as comparisons of functions before symbolic manipulations with CASs and paper-and-pencil; the CASs with operator "|" was used to verify value(s) that make an equation "true" (i.e., solutions to equations). Symbolic transpositions with step-by-step equation solving techniques of applying the distributive

Table 3
Classifying Solution Sets to Linear Equations

Sample Tasks	Techniques	Theory (Concept)
Answer the questions for each equation (A, B, and C): 1. The equation is sometimes, always, or never true. The equation has ____ solutions. I know this because ____. A. Graph of $4x + 1 = 2(2x + 0.5)$ B. Table of $x + 2 = 2x - x + 4$ C. $x + 3 = -x + 3 + x$	• *Translate* from a symbolic equation in x: $ax + b = cx + d$ to graphs and tables of $f(x) = ax + b$ and $g(x) = cx + d$ • *Interpret* graphs as intersecting, parallel, or overlapping • *Classify* solution sets as one, infinite, or zero by interpreting equations in symbols, tables, and graphs. • *Interpret* an equation Exp1 = Exp2 as a relationship between two expressions Exp1, and Exp2.	*1. Solutions to equations can be determined by equality of expressions*. Linear equations are relations between linear expressions that are sometimes, always, or never equal in value. Thus linear equations have one, infinitely many, or zero solutions, respectively.

property, combining like terms, and isolating a variable were introduced on CASs, then paper-and-pencil. Finally, instruction on solving equations with infinite and no solutions followed a similar instructional pattern—translating from symbolic equations to Cartesian graphs and function tables with CASs before symbolic transpositions with paper-and-pencil or CASs. The tasks, techniques, and conceptual learning goals relevant to solving equations are explained next.

Introduced in Table 3, the central idea of Concept 1 is the notion that students must come to understand the equal sign as an equivalence relation in to make sense of solving linear equations in one variable (cf. Knuth et al., 2006). When an equation in one variable is viewed as an equivalence relation it means that the equation is a statement of a relationship between two expressions, $Expression_1 = Expression_2$, that is sometimes, always, or never true; expressions can be compared as functions in which equivalent expressions signify the same function in symbols, graphs, tables, and words (Kieran & Sfard, 1999; Yerushalmy & Gilead, 1997). In the context of solving linear equations, this means an equation has one, infinite, or no solutions, respectively. This view was emphasized by Kieran and Drijvers (2006) in their teaching experiment with tenth-graders in which "the relation between two expressions being equivalent or not, and the corresponding equation having many, some, or no solutions was explored in both CAS and paper-and-pencil tasks" (p. 216). For example, tasks included classifying the truth of solutions, and justifying the number of solutions by examining the equivalence or nonequivalence of expressions related by an equal sign using graphs, tables, and symbols. The central design principle was to support students in linking equivalence of expressions to understanding equations in one variable as equivalence relations before solving equations.

Two additional concepts are introduced in Table 4. Concept 2 is based on the notion that in solving equations from a functions approach, students needed to understand connections among representations to coordinate the meaning of a solution across representation types (Huntley & Davis, 2008; Huntley, Marcus, Kahan, & Miller, 2007). Tasks were designed to support students' techniques of connecting multiple representations, especially in justifying a solution across symbols, graphs, numeric tables, and CAS feedback of "true." For example, a task was to solve an equation in multiple representations then articulate how the solutions were related.

To support the coordination of information across multiple tools including paper-and-pencil and CAS-based representations, students should interpret, reflect on, and make sense of differences among those representations (e.g., Dick & Edwards, 2008). Instruction was designed to support learning experiences that regularly involved the use of both CASs and paper-and-pencil tools. Emphasis was placed on the techniques of translating, transposing and interpreting across and within multiple representations, and reconciling across tools. For example, with Concept 3, students translated from symbolic equations to graphs and tables to predict solutions to equations with no or infinite solutions before establishing the solution set.

Experimentation and ongoing analyses. Authors N. Fonger and M. Rohwer (hereafter referred to as Ms. R) co-planned all lessons. Ms. R taught all lessons, which followed a common pattern: introduction/warm-up problem, check homework, in class activity, summary/exit ticket. N. Fonger led the development of curriculum and instructional design, ongoing and retrospective analyses, and was a participant observer in the classroom, also managing all data collection. After each teaching episode, N. Fonger and Ms. R met for one to two hours to conduct ongoing analyses of the

Table 4
Solving Linear Equations Using Multiple Representations

Sample Tasks	Techniques	Theory (Concepts)
Solve $4x - 9 = -7x + 13$ for x in graphs, tables, and symbols. How does the solution found in graphs relate to the solution found in tables? In symbols? Check using CAS | operator.	▪ Solving by symbolic *transpositions* (e.g., for $4x - 9 = -7x + 13$, add 9 to obtain $4x = -7x + 22$, add $7x$ to obtain $11x = 22$, divide by 2 to obtain $x = 2$). ▪ Solving by *translating* to a graph (e.g., *create* a Cartesian graph of $f_1(x) = 4x - 9$ and $f_2(x) = -7x + 13$, $(2, -1)$ is the point of intersection, *interpret* $x = 2$ is the solution) ▪ Solving by *translating* to a table (e.g., *create* function table for $f_1(x) = 4x - 9$ and $f_2(x) = -7x + 13$, when $x = 2$, $f_1(x) = -1 = f_2(x)$, *interpret* $x = 2$ is the solution). ▪ Solving by *translating* to CAS verbal output (e.g., *evaluate* equivalence on CAS: $4x - 9 = -7x + 13 \mid x = 2$ gives output "true", *interpret* $x = 2$ is a solution) • *Connect* solutions across representation types (e.g., $x = 2$ was "true" on CAS and also the value that makes $f_1(x) = f_2(x)$ on the graph and table).	2. *Different representations can signify the same object.* Different representations/ representation types of the *same* linear expressions, equations, signify the same relationship, pattern, function, or solution set from different yet complementary perspectives. 3. *Solving equations in one variable is conceptualized as a comparison of two functions.* Linear equations in one variable such as $ax + b = cx + d$ for real valued parameters a, b, c, and d, can be solved for the variable x by comparing the functions $f(x) = ax + b$ and $g(x) = cx + d$ for the value of x that makes the equation $ax + b = cx + d$ true. Graphical, tabular, or symbolic methods can be used.
For each Equation (D and E): 1. Use a graph or table to predict if the equation has zero, no, or infinite solutions. Provide a sketch to show why. 2. Solve the equation for the variable. Show your work. 3. Check your solution. Does it make sense? 4. Was your prediction correct? Explain why or why not. D. $6x = 5x + 7 + x$ E. $-s - 4 = -1(s + 4)$	▪ *Interpret* symbolic equations, and graphs and tables of functions to represent infinite, one, and no solutions to an equation (e.g., $6x = 5x + 7 + x$ has no solutions because the graphs $f_1(x) = 6x$ and $f_2(x) = 5x + 7 + x$ are parallel, and the tables never have the same value). ▪ *Transpose* within symbolic representations on paper-and-pencil and CAS to create a series of equivalent symbolic equations (e.g., $6x = 5x + 7 + x$ can be simplified to $6x = 6x + 7$, subtract $6x$ to get $0 = 7$, this equation is never true, thus there are no solutions) ▪ *Interpret* the solution set as the value(s) that make the equation true (if any), Justify the solution using graphs, tables, symbols (e.g., $-s - 4 = -1(s + 4)$ when graphed as $f(s) = -s - 4$ and $g(s) = -1(s + 4)$ is the same graph, so $-s - 4 = -1(s + 4)$ has infinite solutions. Also $-s - 4$ is equivalent to $-1(s + 4)$ so the equation $-s - 4 = -1(s + 4)$ is always true).	

instructional episode using lesson artifacts, and student formative assessment items as data sources. The focus of these meetings was to conjecture and substantiate productive means of support for students' development of representational fluency; N. Fonger summarized these analyses and shared these with Ms. R as a member-checking procedure to improve cycles of design, and future experimentation and analyses. Consistent with CTE method, teaching activities were developed and evolved throughout the experiment to support the evolving understandings of the participants (Cobb, 2000). Our daily analysis of teaching episodes and conjectured instructional supports led to some changes in instructional design, especially in supporting students' interpretation of CAS-based representations. For example, Ms. R reflected that students seemed confused at how to interpret CAS function tables that displayed two functions at once, so we created an instructional aid displaying two tables—each with x and $f(x)$ value pairs—to solve an equation with tables.

Solve for the variable and check your solution using a table, graph, or equation (your choice). Sketch the section of the graph (with the scale marked) or the table and indicate where you found the answer. If you solve the problem using equations, show all of your work.

$t - 2 + 3t = -6 + 4t + 4$ for t

Figure 2. An initial interview task.

Retrospective analyses. During the third phase of the design experiment, N. Fonger conducted retrospective analyses of teaching experiment data. This involved analyses of classroom video and triangulation with analyses of additional data: student work, classroom observation field notes, and reflections from daily meetings between the teacher (Ms. R) and researcher (N. Fonger). The analysis process during this stage followed a constant comparative method (Glaser & Strauss, 1967); with each day taken in succession, conjectures related to a path of reasoning in classroom instruction were confirmed or refuted based on analyses of the aforementioned data sources, which were then tested against the subsequent day's data.

Video data were segmented into critical moments—smaller episodes of instruction that supported or refuted the conjecture instructional theory. These critical moments were coded according to the core concepts and techniques that structured classroom activity in support of representational fluency (as introduced in Tables 3 and 4).

Change in Representational Fluency

Each of Annie, Bryon, and Carlos participated in an individual 50-minute semi-structured task-based interview with N. Fonger at the beginning and at the end of the teaching experiment. Students solved a series of problems involving linear equations with various solution types (i.e., one, none, or infinite solutions). Solving a symbolic equation with infinite solutions is a task type previously identified to pose great challenges for students (e.g., Huntley et al., 2007). The initial and final interview tasks of focus are given in Figures 2 and 3. By design, students had considered similar tasks during the respective pretests and posttests. The focus of the interviews was to discern students' representational fluency in solving these equations. Students were encouraged to "think out loud" as they created, interpreted, and connected representations.

Analyses of video of semi-structured interviews were triangulated with analyses of written and dynamic inscriptions students created with paper-and-pencil and CAS during the interviews. Analyses of each student's strategies were coded per the framework for sophistication in representational fluency (Table 1) and were analyzed in a cyclical fashion across tasks, and interviews. Due to the scope and timeline of this study it was not feasible to track changes in students' levels of sophistication in representational fluency across each day of the teaching experiment. However, careful analyses of Annie's, Bryon's, and Carlos's CASs and paper-and-pencil inscriptions on classroom activities were conducted both (a) during ongoing analyses of teaching experiment data as reflected in daily summaries to document the impact of the curricular and instructional supports on students' change in representational fluency, and (b) during retrospective analyses to garner supporting or refuting evidence of means of support.

Results

This section elaborates the nature of instructional supports for students' change in representational fluency in solving linear equations with CAS/paper-and-pencil.

Instructional Support

Three key means of support are reported: (a) translation from symbolic equations to graphs and tables was

Steps	Algebra
1.) Original problem	$2 - x - x = x + 8 - 3x - 6$
2.) Combine like terms	$2 - 2x = -2x + 2$
3.) Add $2x$ to both sides	$2 = 2$
4.) Subtract 2 from both sides	$0 = 0$
5.) Solution	$x = 0$ and $x = 2$

Andy worked out the problem above. When he substituted his solutions into the original equation, he discovered that both $x = 0$ and $x = 2$ make the equation $2 - x - x = x + 8 - 3x - 6$ true.

a) Was Andy's work correct? Explain why or why not.

b) What is the solution to the equation $2 - x - x = x + 8 - 3x - 6$?

Figure 3. A final interview task.

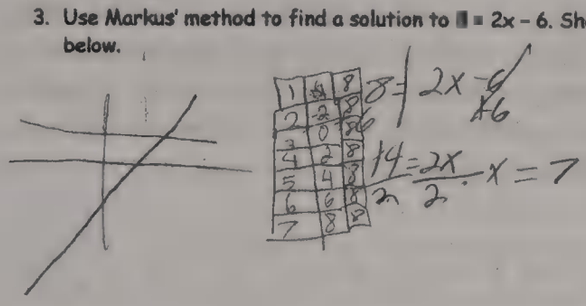

Figure 4. Annie's use of graphs, tables, and symbols to solve a linear equation.

sequenced before symbolic transpositions; (b) solutions were interpreted as invariant features across representation types; and (c) expressions relating an equal sign were compared from multiple representations. Evidence is drawn from analysis of the third section of the instructional sequence focused on equation solving (Table 2, Section 3, Activities 1–4).

Translations from symbols to graphs and tables prioritized. In the teaching experiment, we capitalized on CAS as a representational toolkit to create and interpret graphs and tables. When a functions approach to solving was introduced, instruction emphasized first a translation from a symbolic equation to a Cartesian graph, then a function table before transposing within the symbolic representation type. Annie's equation solving strategies in Figure 4 reflects the typical instructional approach; from left to right, Annie first translated from a symbolic equation to a graph and a function table, then created equivalent symbolic equations.

The approach to translate from a symbolic equation to a graph comparing two functions and corresponding table before formal symbolic manipulations on equations was a continued theme when solving more complicated equations with variables on both sides of the equal sign (of the form $ax + b = cx + d$ with one solution). For example, consider Vignette 1.

Vignette 1. The task is to "Solve the equation for the variable. Show your work. Check your solution. $4x - 9 = -7x + 13$" A typical approach to solving equations was to translate to graphs and tables before performing symbolic transpositions.

Ms. R: "So, if I, I'm not sure by looking at these equations (in reference to $4x - 9 = -7x + 13$), I can graph these in my CAS. What happened to those lines?"

Katrina: "They overlap."

Ms. R: "They cross don't they? They sure are intersecting. Alright. So these guys are crossing. They have a point of intersection. They have one solution. And if I look at my graph can I figure out what value of x they're going to cross at?"

Abila: "Negative one and two."

Ms. R: "So $x = 2, y = -1$. What if I look at the table?"

Davon: "You can check."

Ms. R: "That's the way we do it, too. So if I go to my table, Control T, go to your table, and look where my y-values are the same, OK, that's the only place where the y-values are the same. Right here (gestures to table) $x = 2, y = -1$. That's the only place. So one solution." (CTE, 10/11/11)

Annie's translation from a symbolic to graphic and numeric representations for this task is given in Figure 5a). Annie's paper and pencil work shows that she identified "2" as a "sometimes" solution to $4x - 9 = -7x + 13$ (in Figure 5b). We interpret this as a reflection of the instructional sequence and language used when identifying an equation with one solution as being "sometimes true."

Instruction on how to solve equations with graphs and tables of two functions occurred first. This was followed by instruction on solving equations in a step-by-step symbolic fashion (motivated by a student sharing his work, see Section 3, Activity 1). Next, instruction targeted the coordination of CAS/paper-and-pencil to create a series of equivalent symbolic equations of the form $ax + b = cx + d$ (see Section 3, Activity 2). Finally, the use of CAS techniques to solve equations in a step-by-step fashion to create a series of equivalent equations was introduced.

Solutions are interpreted as invariant features across representations. Connections among representation types were explicitly emphasized when solving equations. For example, we saw evidence of this instructional emphasis in

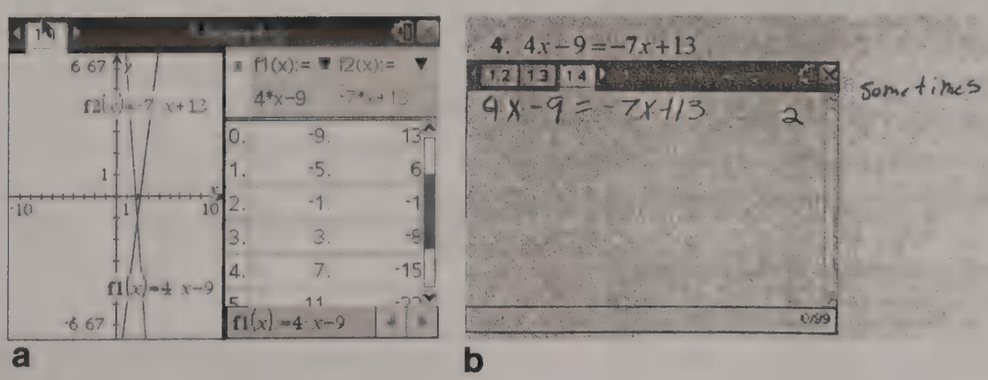

Figure 5. Annie's approach to solving a linear equation with (a) CAS and (b) paper-and-pencil.

the aforementioned work of Annie in Figures 4 and 5. Moreover, in what follows, Vignette 2 illustrates how instructional supports also emphasized leveraging CAS feedback to discern the truth of an equation at a value:

> **Vignette 2.** After identifying the x-value of 26 from the graph and table of $f4(x) = 10 - 3x$ and $f5(x) = 22$, Ms. R introduced a CAS technique of evaluating a symbolic equation at a numeric value to yield a verbal representation of "true," interpreting $x = 26$ as the solution to the equation $22 = 100 - 3x$ (see Figure 6) (CTE, 10/5/11).

Graphic and numeric approaches were also coordinated with solutions determined by symbolic transposition. For example, in reference to the aforementioned functions approach to solving $4x - 9 = -7x + 13$ as in Vignette 1 and Figure 5, Ms. R articulated:

> Now let's look at our other representations (slides CAS screen as in Figure 5a next to symbolic transpositions, not shown) At $x = 2$, (points to x-value of 2 in table, Figure 5a) that was my solution here (points to values of $f_1(2)$ and $f_2(2)$ in table), where these lines crossed (points to $(2, -1)$ in graph, Figure 5a), and we did it symbolically. We did it all three ways. So we got the graph, we got the table and we can see from the rule, that's the same thing as our equation. (CTE, 10/11/11)

From these examples we see that classroom and individual activity promoted an explicit focus on creating and interpreting the solution as an invariant feature of a symbolic equation or pair of functions across graphs, tables, symbols, and CAS feedback of "true."

Comparing expressions related by an equal sign. The approach to comparing expressions related by an equal sign using graphs and tables was followed for equations with infinite and no solutions as well. In Section 3 Activity 3, language of "one solution, infinite solutions, or no solutions" was used to parallel and build on the approach to comparing expressions related by an equal sign in Section 3 Activity 2 with the language of "sometimes, always, or never true." Students were often given choices for which representation type to use to justify their solutions. For example, Annie's work often demonstrated her preference to describe the nature of a solution set by interpreting graphic representations. For example, she crossed her hands like an "X" explaining that a solution was "where both of them, kind of like, intersect... this is that sometimes thing." Other students, such as Bryon, often chose to perform symbolic transpositions. Finally, in Vignette 3, instruction targets symbolic equivalence of expressions.

Figure 6. A solution to an equation evaluates as "true" on CAS.

> **Vignette 3.** Ms. R: "Infinite solutions. Why? Why is it infinite solutions? Cause it means the same thing on either side of the equal sign. Look back at this step right here ($-r - 5 = -r - 5$). What do you see about the expression on the left and the expression on the right?"
>
> Ethan:"They're the same."
>
> Ms. R: "They're exactly the same. So you have equivalent expressions. If your expressions on either side of the equals sign are exactly the same that means you have the same line, you have exactly the same line, so infinite solutions." (CTE, 10/13/11)

In the above exchange the teacher focused students' attention on interpreting equivalent expressions related by an equal sign as being the same across symbolic and graphic (i.e., "same line") representation types. Also, the equation was classified as having "infinite solutions." We note that the instructional strategies in Vignettes 1 and 3 provided few

Table 5
Students' Change in Representational Fluency: Solving a Linear Equation with Infinite Solutions

	Sophistication	
	Lesser	Greater
Nature of Students' Discursive Activity	Student expresses an incorrect or incomplete creation, interpretation, or meaning.	Student expresses correct and complete creations, interpretations, and meanings.
Interview	SOLO Level	
	Prestructural	Unistructural
Initial	A B C A B B B	
Final	A B C C B	A
	Multistructural	Relational
Initial	A B A B A	
Final	B B B B	A A A B

A single letter is used to denote a strategy for each student (A = Annie, B = Bryon, C = Carlos). Multiple letters indicate multiple strategies for a given student.

opportunities for students to explain their thinking and to reflect on and justify their work; an issue we discuss later.

Students' Change in Representational Fluency

In light of the results of the instructional context, we now turn to an account of students' change in sophistication of representational fluency. Table 5 summarizes results of students' sophistication in representational fluency in solving the tasks given in Figures 2 and 3. In the initial interview Annie and Bryon both demonstrated prestructural and multistructural levels of representational fluency, while by the final interview Annie demonstrated unistructural, and both Annie and Bryon demonstrated relational levels. Carlos persisted at the prestructural level.

At the onset of the teaching experiment, all three students were characterized to have low levels of sophistication in representational fluency in solving equations with incorrect creations and/or interpretations of representations. Consider Annie's work in Vignette 4.

Vignette 4. Annie attempted to solve $t - 2 + 3t = -6 + 4t + 4$ for t. Annie was initially stuck within the symbolic representation type, then N. Fonger asked Annie if she could compare $t - 2 + 3t$ and $-6 + 4t + 4$ as a graph (of functions of t). Annie created graphs using her graphing calculator and paper-and-pencil (shown in Figure 7a, b).

Annie reflected, "How they (Figure 7a, b) equal the same thing I have no idea."

N. Fonger prompted, "So if you were to write down what's the solution to that equation, what would you write down?" Annie replied, "I have no idea what the solution would be. The solution's like, it's the same thing, it's the same thing, it should probably equal the same thing in graph and in table, but right here (points to her symbolic work that shows $7t + -4$), it doesn't actually equal to the same thing."

Annie's final strategy was to work within the symbolic representation type, making a computational mistake in adding like terms (see Figure 7c). This work led Annie to conclude that $t = -4$ was *the* solution to the equation (Initial Interview, 9/13/11).

Like Annie, Bryon and Carlos demonstrated inconsistent and incorrect strategies for combining like terms in symbolic equations. For Bryon, he had initially been successful in combining like terms with paper and pencil to reach the equation "3T = 3T." From this equation he concluded, "Oh, I think I messed up somewhere in there." Thus despite a correct creation of an equivalent equation, he was not able to draw meaning about the solution to this equation, a prestructural level of fluency. Bryon followed this with an additional strategy in which he incorrectly combined like terms to reach 0 = 2T, concluding, "I figured that all the *t*'s are zero." Carlos used his graphing calculator to add the numbers in $t - 2 + 3t = -6 + 4t + 2$ because "that's what my eighth-grade teacher told me—you don't have to put a bunch of variables and letters in there; you just add the numbers." He concluded the equation was "false"; one equation can't equal another.

With respect to multiple representations, both Annie and Bryon were able to graph two lines on their calculator when prompted, yet the "sameness" they observed was not yet meaningful with respect to the solution to the equation. The ability to correctly create representations, moving across representation types, without a correct interpretation of the meaning is a multistructural level of representational fluency.

In the final interview, Annie was initially unable to make sense of Andy's work within the symbolic representation type (a fictional student, as given in the problem statement), yet leveraged graphs and CAS feedback to make progress as seen in Vignette 5.

Vignette 5. Without explicit prompting, Annie decided, "If I take both of them (points to $2 - 2x$ and $-2x + 2$) which is probably going to be equal in the graph, it may help (creates graph, Figure 8a). They have (creates table, Figure 8b) infinite solutions (cursors in table from $f6(0) = 2$ to $f7(0) = 2$)."

Annie later decided to insert a CAS calculator page in which she evaluated the original equation at the values of $x = 0$ and $x = 2$ [Figure 9]). She claimed "They're both true . . . each one is correct."

Prompted to explain to someone else what the solution was, Annie replied, "It could be either zero, or two."

With further probing if the solution "could it be anything else," Annie replied, "Yes, including that it's a very infinite solution I believe. (Switches back to split graph and table view on CAS like Figure 8, and scrolls down to $f7(3)$) . . . Oh I get what they're doing (points to $2 - 2x = -2x + 2$ and $2 = 2$), they're adding $2x$ to both sides . . . Oh I get it, because it's infinite solution. They're both the same, except for with minuses. . . . Yeah, they're infinite solutions it's hard to tell what x equals." (Final Interview, 10/18/11).

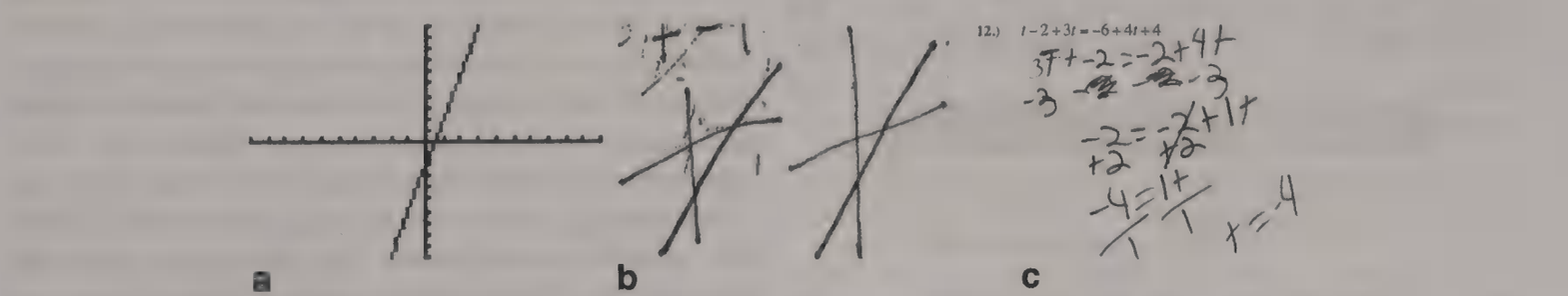

Figure 7. Annie's (a) graphing calculator graph, (b) paper-and-pencil graphs, and (c) symbolic equation solving activity during the initial interview.

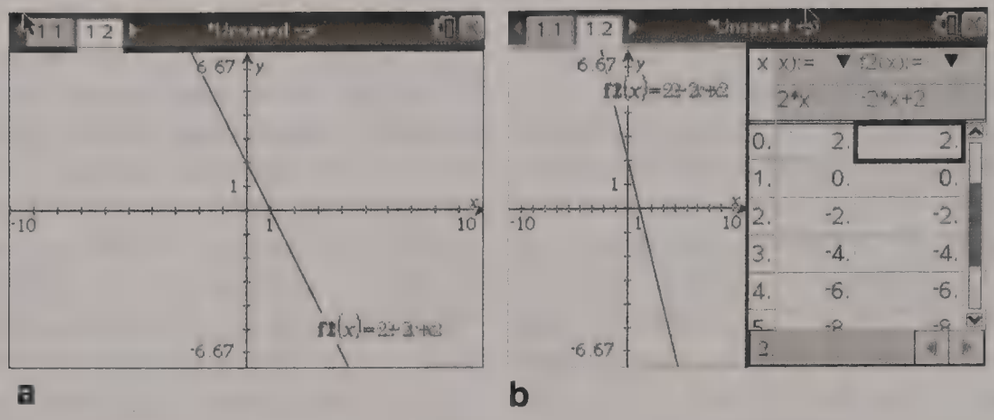

Figure 8. Annie's (a) graphic and (b) split screen graphic/numeric equation solving activity during the final interview.

Annie was successful in drawing meaning about the solution set to the equation from graphs and tables, and coordinating the meaning of $x = 0$ and $x = 2$ across the symbolic equation and CAS feedback of "true." When she returned to the symbolic equation, she recognized the "sameness" in the expressions related by an equal sign, which meant infinite solutions. One way to interpret Annie's last statement, "It's hard to tell what x equals," is the meaning of "infinite solutions" is not well connected to making statements about the solution set, which is commonly expressed in the form of a statement that "$x =$ __" (a number). Annie's meaning for "infinite" solutions was conveyed when interpreting graphical, numerical, and symbolic representation types, in addition to the CAS feedback of "true," yet conventional language about the solution set as being all real numbers was not evident, a point of confusion for Annie. Like Annie, Bryon demonstrated change in representational fluency at the relational connection level by using CAS feedback to confirm $x = 2$ is true, concluding, "It is a solution, but I don't know how." After creating CAS graphs of $f3(x) = 2 - x - x$ and $f4(x) = x + 8 - 3x - 6$ he noted the graphs "overlapping the whole time" yet insisted the solution was "zero equals zero [0 = 0]" without recognizing infinite solutions (a multistructural level).

In contrast to Annie and Bryon, Carlos did not make progress beyond the prestructural level. In reading the problem statement, Carlos did not think it was possible to have two solutions. He expressed confusion about the problem statement, "Since the x equals zero and x equals two, because if x equals zero how can x equal two?" Prompted to use his CAS, he typed "0" Enter, then "2" Enter. He did not interpret these as solutions to equations.

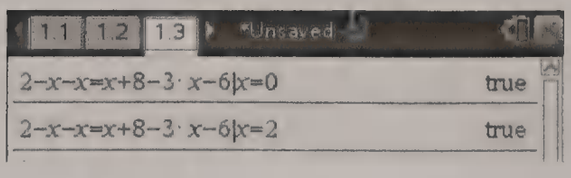

Figure 9. Annie's symbolic to verbal translations on CAS to confirm solutions to a linear equation during the final interview.

Discussion

Supporting students' representational fluency and meaningful equation solving in technology-rich settings is an important and timely issue (Arbaugh et al., 2010; NRC, 2001). In this exploratory study we employed a learning progressions approach to elaborate both means of support for student learning and the nature of students' representational fluency. We found instructional supports to be productive for some students: (a) translation from symbolic to graphic and/or numeric representation types was prioritized before symbolic transpositions in solving equations, (b) connecting solutions across CAS and paper-and-pencil inscriptions including CAS feedback of "true," and (c) the treatment of the equal sign as a relational symbol in comparing expressions that are sometimes, always, or never true. Evident throughout the CTE, and as measured in students' equation solving activity in interviews, both Annie and Bryon advanced to more sophisticated levels of representational fluency in solving a linear equation with infinite solutions both within and across representation types, while Carlos's representational fluency did not change. This finding has implications for research on how students' sophistication in representational fluency is characterized. In-depth examinations of students' problem solving may reveal nuances between levels of sophistication that shed light on how students' understandings of mathematical ideas are intertwined with their representational fluency (e.g., see Fonger, Dogan, & Ellis, 2016). While levels above relational were not observed in this study, prior research has shown that older

students demonstrate sophisticated representational fluency in solving equations with CAS (e.g., Fonger, 2011). This stresses the idea that developing sophisticated fluency occurs over multiple years, not a single teaching unit.

Instructional Supports for Representational Fluency

Across these data we saw how instructional support and student activity can embody the core concept of an equation as a relationship between two expressions across graphs, tables, and symbols. For students such as Annie, we conjecture the comparison of expressions via multiple representation types throughout the teaching experiment was developmentally important, especially how the language of "sometimes, always, never true" was linked to language of "one, infinite, or no solutions." Second, the instructional approach of comparing graphs and tables before symbols seemed significant in Annie's choice to use graphs to identify solutions/solution sets to equations, yet not all students took up this strategy, nor in a meaningful way. For example, Carlos did not use multiple representations in the final interview, and while Bryon was able to create graphs from a one-variable equation he did not interpret the meaning with respect to solving the equation. Thus for some students, this study corroborates existing research findings suggesting the potentially meaningful sequence of graphic and numeric approaches to solving equations prior to symbolic transpositions (e.g., Kieran & Sfard, 1999; Yerushalmy, 2006; Yerushalmy & Gilead, 1997). Yet for other students, this instructional approach was not supportive. Third, the instructional design emphasized the use of the CAS feedback of "true," which was highlighted in the final interview as a key way that Annie and Bryon demonstrated connections across representation types. The role of coordinating the CAS feedback of "true" or "false" with graphic and numeric approaches to solving equations resonated with students. However, for equations with infinite solutions the truth of one or two values needs to be situated with respect to an infinite solution set, a difficulty of solving this type of equation.

The use of natural language to describe equivalence of "sometimes, always, or never true" in graphs seemed meaningful to students throughout instruction and in the final interviews. The use of CAS feedback of "true" for evaluating solutions seemed particularly supportive of students' problem solving. This finding underscores the importance of addressing students' representational fluency with natural language and technology feedback in the complex domain of mathematical equivalence and equation solving from a functions approach.

Improving Future Instructional Design

To build upon this exploratory study, several cycles of experimentation are needed to refine instructional designs, and garner empirical support for effective instruction linked to students' development of representational fluency. In light of the finding that Carlos did not change his representational fluency, it is feasible to infer that the approach to sequencing and connecting multiple tool-based representations to understand equations as equivalence relations was not an effective instructional approach for him. In light of research on effective classroom interaction patterns (cf. Wood, 1998), we hypothesize that students in this study may have achieved greater sophistication in representational fluency had they been provided more consistent opportunities to explain their reasoning and discuss their own meaningful connections. Additionally, there is a need to better understand the nature of instructional supports for diverse populations of learners including how to cultivate effective student-tool relations (e.g., Kieran & Drijvers, 2006). How might CAS be further leveraged as a "partner" in students' learning?

Finally, we wonder how other noncognitive dimensions of the classroom environment might better support students' learning. For instance, Carlos, like some other students in the class, had a relatively disengaged disposition during his day-to-day activity during the CTE, and seldom completed the in-class activities. Future instructional designs should attend to students' motivation to solve problems with multiple representations. For example, how might a story, metaphor, or embodied approach support students to engage in meaning-making? How might task design and instruction support the "need" for bolstering representational fluency? In close, this research extends studies that focus on students' difficulties in translating between representations to solve problems alone. Examining links between individual activity and viable instructional supports in complex learning environments is an important issue that warrants further investigation in research and practice.

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[1]This school site was chosen based on access through an existing partnership between the school and a local university, and the opportunity to work with a collaborating teacher who volunteered.

[2]All student names are pseudonyms.

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Impact of Middle School Student Energy Monitoring Activities on Climate Change Beliefs and Intentions

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The Going Green! Middle Schoolers Out to Save the World project aims to direct middle school students' enthusiasm for hands-on activities toward interest in science and other STEM areas while guiding them to solve real-world problems. Students in this project are taught by their teachers to use energy monitoring equipment to audit standby power consumed by electronic devices in their homes and communities. Major findings were: (a) Beliefs in climate change increased more for students in the treatment than comparison group, pre to post; and (b) For girls there was a larger positive impact on climate change beliefs than for boys. These and additional findings presented in this paper provide evidence that a hands-on engaged-learning curriculum can have a positive influence on climate change beliefs and intentions and strengthen the association between the two constructs.

While a majority (63%) of American adults report they believe in the existence of climate change, few (14%) say they are "very worried" about it (Leiserowitz, Maibach, Roser-Renouf, Feinberg, & Howe, 2013). Those surveyed see climate change as a distant threat that will impact future generations (Leiserowitz et al., 2013). Americans have been shown to be more skeptical than people in other countries (Carlsson et al., 2010; Infographic, 2011). A European study of 27 member states of the European Union (EU) revealed an overall belief that climate change is a serious issue (Eurobarometer, 2009). Researchers have pointed out a lack of curriculum content regarding climate change in schools (Choi, Niyogi, Shepardson, & Charusombat, 2010), leading to students receiving more of their information from the media than teachers (Robertson & Barbosa, 2015). The age-stability theory assumes that adolescents' social and political attitudes are already strongly developed by the time they leave secondary school (Alwin & Krosnick, 1991; Sears & Funk, 1999). Therefore, the development of positive environmental attitudes in childhood is an important element in shaping behaviors in later life (Ballantyne, Connell, & Fien, 2006; Chawla, 1999; Meinhold & Malkus, 2005). Researchers found a positive relationship between learning about the environment at school and the level of environmental concern (Barth & Michelsen, 2013; Hungerford & Volk, 1990; Knafo & Galansky, 2008). Researchers have concluded that educating students about the environment can influence their future behaviors. Few studies have attempted to measure beliefs and intentions regarding climate change at the middle school level.

Literature Review

Theoretical Background

Intentions and beliefs have been recognized as important in science education for decades (Haney, Czerniak, & Lumpe, 1996). In the theory of planned behavior (Ajzen, 1985), beliefs are used to predict an individual's intention to engage in behavior. Behavioral intentions have been defined as the indication of an individual's readiness to perform a given behavior, based on attitudinal beliefs and perceived behavioral control (Ajzen, 2002), and are assumed to be an immediate predecessor to behavior. The theory of planned behavior was developed from the expectancy value models (Ajzen & Fishbein, 1980) where if people evaluate a suggested behavior as positive (attitude) there is a higher intention (motivation) to perform the behavior. Many studies have confirmed a high correlation between attitudes and behavioral intention and subsequently with behavior (Sheppard, Hartwick, & Warshaw, 1988). One study measuring attitudes toward human-induced climate change found students who have a more accepting attitude toward climate change are more likely to express a willingness to take action (Sinatra, Kardash, Taasoobshirazi, & Lombardi, 2012).

Fortus (2014) pointed out the lack of published research during the first decade of the 21st century related to the affective domain in science education. He further contended that very little learning occurs without a desire to learn (motivation, attitudes, beliefs, self-efficacy, interest, self-concept, and others):

> Regardless of whether students will go on to work in a STEM-related profession or just live in a STEM-

> influenced world, we should strive for all to have positive attitudes to science and its role in society, motivation to understand the science of issues directly related to their lives and their general well-being, and a belief in their ability to make sense of issues. (p. 822)

Measuring students' attitudes toward science-related educational endeavors allows educators to plan appropriate instruction and determine effectiveness of the curriculum.

Climate Change Engagement

A comprehensive, international review of climate change engagement for young people revealed many studies that captured youth views on climate change but little research on ways in which this population could be more effectively engaged (Corner et al., 2015). Scholars agree that how individuals understand climate change is important in their willingness to take action (Lorenzoni & Pidgeon, 2006). However, much research has also shown that knowledge about climate change does not necessarily lead to change in behavior (Gardner & Stern, 2002; Kahlor & Rosenthal, 2009). Some type of intervention may be necessary to propel individuals from beliefs to behavioral changes due to the gap between knowing about climate change and changing behavior to make a difference (Blake, 1999; Kollmuss & Agyeman, 2002). Redman and Redman (2017) found subjective knowledge (values, attitudes, and beliefs) to be the most important type of knowledge for an educational intervention in achieving behavioral goals toward sustainability. The affective dimension of climate change is believed to provide a critical link between knowledge and attitude versus action (Leiserowitz, 2006; Sundblad, Biel, & Garling, 2007; Weber, 2010). There may be a disconnect between attitudes, intentions, and actions due to inadequate practical knowledge on how to individually reduce emissions (Wolf & Moser, 2011). Individuals need specific information on ways in which they can realize changes in their energy consumption (Wolf & Moser, 2011). When students feel like the issue of climate change is serious but solvable, there is a positive association with pro-environmental behavior (Ojala, 2012).

> A substantial amount of international research also demonstrates that children and young people are more likely to understand, care and act on climate change if they can engage with it directly and experientially, through some form of educational, outreach, or social activity. (Corner et al., 2015, p. 528)

Therefore, it is important to introduce the issue of climate change to students through hands-on, real-world learning experiences.

Hands-on Active Learning

The strategies used for active learning closely align with guidelines employed by the National Research Council to develop the Next Generation Science Standards (NGSS) (National Research Council, 2012). Inquiry-based science was highlighted by the NGSS as the core of teaching and learning for students to develop knowledge and understanding of scientific ideas including an understanding of how scientists view the natural world. The term, inquiry-based science, was derived from "hands-on," "minds-on," and "scientific inquiry" through iterations of science education standards (National Academies Press, 2013).

Student-centered active learning has been shown to improve long-term knowledge retention and deep understanding (Akinoglu & Tandogan, 2007; Bonwell & Eison, 1991; Gallagher, 1997) as well as fostering creative and problem-solving skills (Shieh & Chang, 2014). Hands-on learning has been shown to positively impact student understanding of science concepts (Costu, Unal, & Ayas, 2007) more effectively than traditional instruction (De la Hozi Casas & De Blas del Hoyo, 2009; Sadi & Cakiroglu, 2011). Studies have also shown hands-on learning activities to improve students' understanding of climate change (Karpudewan, Roth, & Abdullah, 2015; Taber & Taylor, 2009). When using the active learning approach, education becomes more personally meaningful and takes advantage of students' natural curiosity. This approach prepares students for the future by having students communicate, collaborate, and try new approaches in finding solutions to real-world problems. A study to find best practices for teaching middle school science found that one of the best practices is hands-on activities that were often in the form of guided science inquiries (Oliveira et al., 2013). Research has also shown these types of hands-on, active learning strategies increase middle school-aged girls' interest in science as well as have an impact on their ideas about participating in a science-related career (Kim, 2016).

Middle school is an appropriate age to develop an interest in science that will persist through secondary school, into college and beyond into a career. Providing authentic, active learning experiences contributes to the internalization of learning about science (Christensen & Knezek, 2015a). Teenage years are a good time to introduce the impact of climate change when views are still being formed (Corner et al., 2015).

Active learning principles are rooted in Dewey's "learning by doing and experiencing" principle (Dewey,

1938). Dewey advocated that a child's schoolwork should have meaning and be engaging as well as have connections to other disciplines and life experiences. The curriculum used for the participants in this study includes hands-on learning activities in which students engage in finding solutions to a real-world problem—climate change.

The purpose of the research study reported in this paper was to confirm the appropriateness of behavioral theories used to predict an individuals' readiness to perform a given behavior (Ajzen, 1985) to the specific context of climate change beliefs and intentions. Through a curriculum that provides a comprehensive study of the issues related to climate change and tools to see how they can make an impact on the reduction of standby power, students can realize specific ways in which they can make a change in the environment. Learning principles related to active, hands-on learning applied to student beliefs and intentions regarding climate change led to hypothesizing an impact in this context.

Research Questions

This study examined pre-post changes in climate change beliefs and intentions among middle school students participating in hands-on curriculum activities related to energy and climate. Three specific research questions were addressed:

1. To what extent do middle school students who participate in hands-on energy monitoring/greenhouse gas projection activities become more positive in: (a) beliefs that climate change is happening in our world, and (b) intentions to take actions that make a difference in reducing contributors to climate change?
2. What gender differences exist for the effects of participating in hands-on energy monitoring curricular activities, regarding student beliefs about the existence of climate change and their intentions to take actions to improve the environment?
3. To what extent are climate change beliefs related to climate change intentions?

Methods

Instrumentation

The climate change attitude survey (CCAS) was used in this study to measure middle school student beliefs about climate change and also their intentions to enact positive change. The 15-item survey contains Likert-type items rated on a 5-point scale from Strongly Disagree (1) to Strongly Agree (5). CCAS was developed by the authors based on adaptations from other climate-related surveys intended for different audiences or different purposes (Champeau, 1997; Leiserowitz et al., 2013). The instrument viewed as a whole was found to be reliable for middle school students (Cronbach's Alpha = .89 for 15 items) and construct validity was determined through factor analysis (Christensen & Knezek, 2015b). Two constructs were represented among the items: Beliefs and Intentions (Christensen & Knezek, 2015b). The current study's set of data revealed respectable reliability as measured by Cronbach's Alpha (.90 for Construct 1 and .78 for Construct 2). Construct 1 consisted of 10 items related to beliefs and included items such as "I believe there is evidence of global climate change." Construct 2 consisted of five items related to intentions, such as "We cannot do anything to stop global climate change" (reversed).

Participants

The CCAS was completed by middle school students in grades 5–8 participating in the Going Green! Middle Schoolers Out to Save the World (MSOSW) project. For the analyses presented in this paper, pre-post data were collected from 794 treatment and 521 comparison students from 18 treatment classrooms and 7 comparison classrooms. These students represented schools in the states of California, Hawaii, Louisiana, Maine, Michigan, North Carolina, Texas, Vermont, and Virginia. The school locations were selected for their diversity in climate zones, rural versus urban locations, and public versus private funding status. The gender representation included 53.6% boys and 46.4% girls. The largest percentage of students (47%) were in 6th grade due to the better fit of the curriculum to that grade level in most states. Students were asked to select the ethnicity to which they most identified and the data included 50.7% Caucasian, 14.6% Asian, 12.8% Hispanic, 7.4% African American, 3.5% Native American/Pacific Islander, and 11.1% Other. Fifty-two percent of the students represented urban areas while 48% represented rural schools. The majority of students were from public schools (89%). Regarding climate zones, students were from 6 of the 12 different climate zones in the United States: East North Central, Northeast, South, Southeast, West, and South Pacific. These demographic data, along with other attitudinal and content data related to STEM and demographic items, were gathered through an online server at the beginning (pre) and end (post) of the 2015–2016 school year, as part of the project's annual data collection activities. Teachers who attended the summer institute received instructions on administering the surveys while all teachers were sent instructions via email from project personnel. Students either went to a school lab or had access to individual laptops to submit data under the supervision of their teachers.

About the Project

The Going Green! MSOSW project aims to direct middle school students' enthusiasm for hands-on activities toward long-term interest in STEM while guiding them to solve real-world problems. The study included both treatment and comparison groups of students. Treatment classrooms were selected by inviting teachers via colleagues in multiple locations from different climate zones in the United States. Comparison classrooms were selected from those who expressed interest in the project but were not initially included due to limitations of numbers of classrooms or the classroom teachers were invited as similar comparisons of those who participated. The goal was to enhance curriculum topics that were already in place as opposed to an "add-on" to curriculum. The comparison classrooms collected pre-post data but taught the normal curriculum on climate change provided by their schools, which followed their own state or national standards. Beginning in the year after they provided comparison data, teachers were invited to be treatment classrooms for the project.

In the MSOSW project, treatment classroom teachers attended a two-day summer institute to learn about the energy/climate curriculum and how to implement the hands-on activities with their students. MSOSW teachers were also provided with classroom sets of 30 energy monitors, web enhanced teaching resources, the curriculum itself, and on-going support from the project personnel. Students in this project are taught by their teachers to use energy-monitoring equipment to audit standby power consumed by electronic devices in their homes and communities. The focus of the three-week curriculum and activities is standby power, which is power that is being used by appliances when they are plugged in but serving no useful function, and how this waste contributes to greenhouse gas emissions and global climate change. The meters used for the monitoring activities can be adjusted for local electricity costs and have single-button calculations of the connected appliances' consumption of kilowatt hours, cost per month, and CO_2 released into the atmosphere while generating the electricity consumed in standby mode.

Standby power is the electricity consumed by many appliances when they are plugged in but "turned off" (U.S. Department of Energy, 2011). The U.S. Department of Energy has estimated that over the lifetime of a typical home appliance, 70% of the power consumed will be when the appliance is turned off (U.S. Department of Energy, 2011).

The curriculum includes hands-on, engaging activities in which the goal is to have students understand and internalize the impact they can individually and collectively have on the climate by doing simple things like unplugging devices in their homes and schools that were plugged in but serving no useful function (standby power). The activities included researching and comparing types of renewable and non-renewable energy sources used to power their electricity, hands-on measurement of energy consumption, greenhouse gas experiments, and using a simulated online game environment. Each of the lessons in the unit includes a list of the educational standards addressed, such as the NGSS. For example, in lesson two when students are identifying evidence and possible causes of climate change, they are addressing science as inquiry asking questions to clarify evidence of the factors that have caused the rise in global temperatures over the past century (NGSS MS-ESS3–5).

During MSOSW project activities, middle school students learn to measure the standby power consumption of various appliances in students' homes. Students produce a home inventory of appliances they believe might be consuming standby power, then take an energy monitor home after being trained by their teacher. The students monitor home appliances, record data, and then take their recorded measures to school where the observations of all students are combined into a spreadsheet projection model produced under the guidance of the teacher. After measuring standby power, students aggregate their data for spreadsheet projections to explore energy conservation plans that could lower a family's monthly electric bill and reduce the greenhouse gas emissions that contribute to global climate change. Classes use the spreadsheet models to perform "what if" projections of the effect of unplugging vampire (standby) power consuming devices, and determine the effect of their findings on the changing climate in our world. Students share their results with other participating middle school students from across the United States by contributing to an online project wiki in which other classrooms in the project can see and compare their findings to other classrooms participating in the project. Students also shared their public service announcements on the project wiki.

Data Analysis and Results

Pre-Post Changes in Climate Change Beliefs and Intentions

The CCAS was used to assess two constructs regarding climate change—beliefs in climate change and intent to take action based on those beliefs. One-way analysis of variance (ANOVA) was conducted to compare differences between treatment and comparison groups at pretest and at posttest for the two constructs. Additional ANOVAs were

Table 1
Total Climate Change Survey, Pre-Post, Treatment versus Comparison

		N	Mean	*SD*	Sig.	ES
Treatment	Pre	792	3.65	.54		
	Post	733	3.86	.56	.0005	.38
Comparison	Pre	508	3.56	.57		
	Post	521	3.66	.60	.005	.17

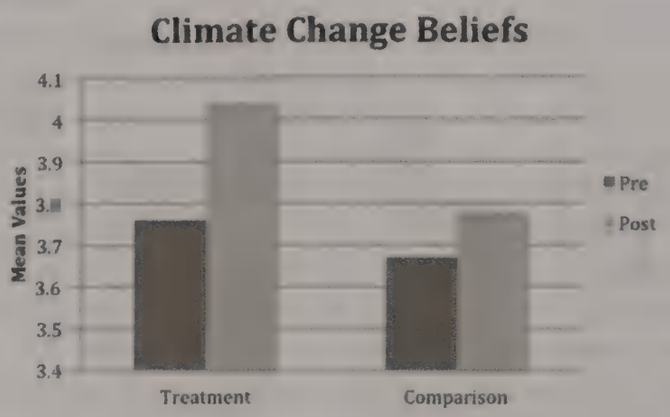

Figure 1. Climate change beliefs, pre-post, treatment versus comparison students. [Color figure can be viewed at wileyonlinelibrary.com]

conducted to compare girls and boys at pretest and posttest. Pearson *r* correlation analyses were conducted to determine the relationship between the climate change constructs between pretest and posttest time for both treatment and comparison participants.

Table 1 shows the treatment and comparison pre-post means for all 15 items on the survey. One-way ANOVA was used to compare pretest to posttest for both treatment and comparison students. Effect sizes were calculated to appraise the educational significance (Bialo & Sivin-Kachala, 1996) of the findings in addition to the statistical significance.

Regarding Research Question #1, while both treatment and comparison groups had significant increases ($p < .05$), the increase was stronger for the treatment group as is also shown by the effect sizes for each group (ES = .38 for treatment and ES = .17 for comparison participants).

For the 792 students in the treatment classrooms across all grade levels, there were significant ($p < .05$) positive changes in both their beliefs ($p < .0005$) (ES = .43) and intentions ($p < .05$) (ES = .20) to make changes to help address climate change issues (see Table 2). For the 508 students in the comparison classrooms, there were also significant increases in positive beliefs ($p = .02$) (ES = .15) from pretest to posttest time periods. For intentions, the comparison group gains were significant ($p < .05$) as well ($p = .046$) (ES = .21) (Table 2). As shown in Figures 1–3, the gains for the treatment group in climate change beliefs (ES = .43) were much larger than for the comparison group (ES = .15), with the treatment group gaining significantly ($p < .05$) more than the comparison group regarding climate change beliefs. There were no significant differences between the treatment and comparison groups for gains in climate change intentions.

Disaggregated Findings by Individual Items

Because of the focus of the project on specific actions that students could take to reduce CO_2 production and its impact on our climate, an item analysis of the CCAS was conducted. For the treatment group all of the individual items increased (binomial $p < .0001$) with 12 of 15 items having significant ($p < .02$) increases individually. For the comparison group, 10 of the 15 items increased but this occurrence was not found to be rare by chance (binomial $p = .15$). The three of the 15 items that had significant ($p < .02$) increases pre to post for treatment students but not for comparison students were: "I am concerned about global climate change"; "The actions of individuals can make a positive difference in global climate change"; and "I think most of the concerns about environmental problems have been exaggerated (reverse coded)." The one item that had a significant ($p = .03$) increase for the comparison group but not the treatment group was "It is a waste of time to work to solve environmental problems" (reverse coded).

Three additional items were added to the original two-construct survey instrument to pilot test their abilities to enhance the original constructs. Of the three items, one showed significant differences between the groups and was

Table 2
Pre-Post Differences for Treatment and Comparison Students for Two Climate Change Constructs

		Treatment					Comparison				
		N	Mean	*SD*	Sig.	ES	*N*	Mean	*SD*	Sig.	ES
Pre	Climate change beliefs	792	3.76	.67			508	3.67	.70		
Post		733	4.04	.63	.0005	.43	521	3.78	.76	.021	.15
Pre	Climate change intentions	792	3.48	.75			508	3.37	.76		
Post		733	3.63	.79	.0005	.20	521	3.53	.74	.001	.21

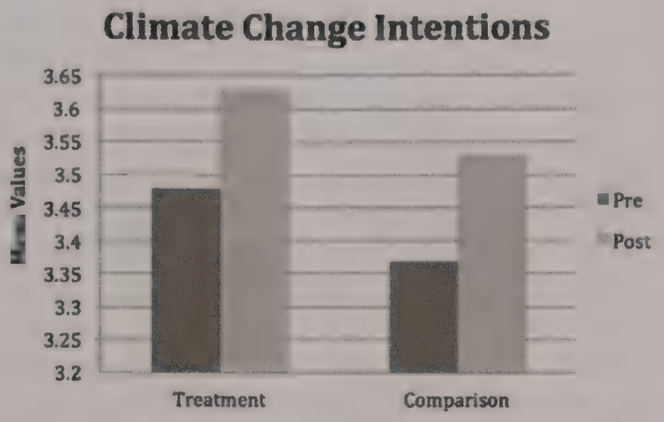

Figure 2. Climate change intentions, pre-post, treatment versus comparison students. [Color figure can be viewed at wileyonlinelibrary.com]

a significant discriminator for the treatment versus comparison group. For the item, "Environmental problems can be solved without big changes to our way of life," there was a large difference between treatment students and comparison students. Students who participated in the standby power monitoring activities increased significantly ($p = .03$) while the comparison students actually decreased (although NS). The other two additional items showed no significant differences between treatment and comparison students.

Effect of Gender on Climate Change Beliefs and Intentions

Additional analyses were conducted on the data disaggregated by girls and boys to determine whether changes in beliefs and intentions attributable to project activities differed for boys and girls. While both climate change beliefs and intentions were examined for pre-post gains, there were only significant differences ($p < .05$) for climate change beliefs. As shown in Table 3, the treatment group girls gained significantly ($p = .009$, interaction term $f = 6.81$, $df = 1$) more than the comparison group girls on climate change beliefs. The magnitude of the effect of

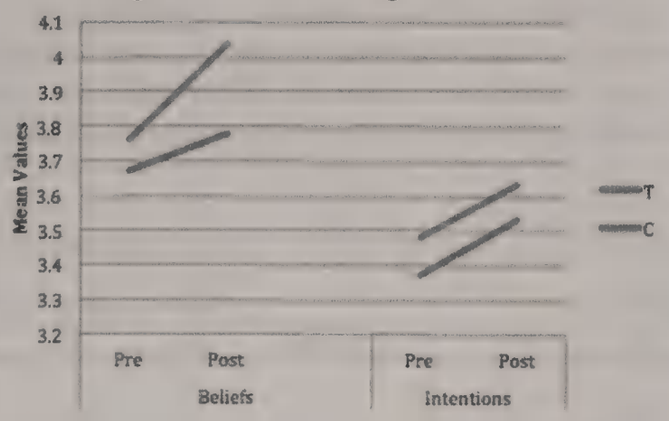

Figure 3. Treatment versus comparison pre and post for climate change beliefs ($p < .05$) and intentions (NS). [Color figure can be viewed at wileyonlinelibrary.com]

Table 3
Treatment/Comparison by Pre-Post Means Regarding Climate Change Beliefs for Girls and Boys

	Girls			Boys		
	N	Mean	*SD*	*N*	Mean	*SD*
Treatment pretest	351	3.73	.60	441	3.78	.72
Comparison pretest	251	3.69	.68	257	3.66	.71
Treatment posttest	331	4.04	.58	402	4.04	.67
Comparison posttest	256	3.80	.66	265	3.75	.71

MSOSW treatment group activities over the comparison group was moderate (Cohen's $d = .33$) and educationally meaningful according to established guidelines (Bialo & Sivin-Kachala, 1996). As shown in Table 3, for boys, the effect of MSOSW activities on climate change beliefs for the treatment group was also significantly ($p = .05$, interaction term = 3.82, $df = 1$) greater than for the comparison group. The magnitude of the effect of treatment versus comparison was small (Cohen's $d = .22$). As shown in Figures 4 and 5, these findings collectively indicate that the positive effect on girls was stronger than the effect on boys regarding climate change beliefs. These findings align with results in prior MSOSW project years regarding gender differences–that the effects were especially strong for girls (Knezek, Christensen, Tyler-Wood, & Gibson, 2015).

Relationship of Climate Change Beliefs to Intentions

Additional analyses were conducted to examine the relationship between climate change beliefs and intentions (RQ3). As shown in Table 4, at the time of the pretest for treatment students, climate change beliefs and intentions were significantly correlated ($r = .307$, $p < .0005$) with a moderate strength of relationship (Cohen, 1988; Lenhard & Lenhard, 2016). Nine percent (9%) (.307 × .307) of their variance was in common. Also shown in Table 4, for the treatment group at the time of the posttest, the strength of

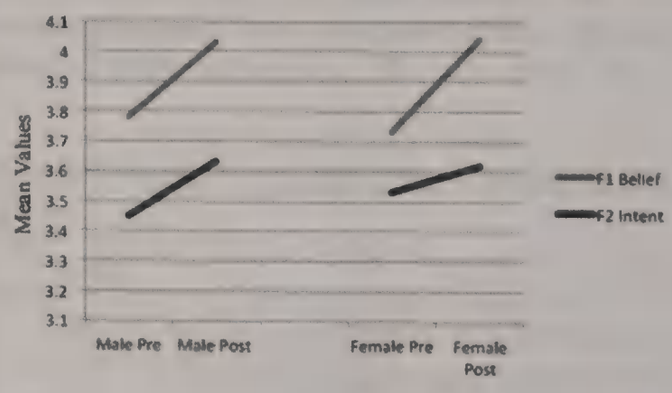

Figure 4. Treatment students male versus female for climate change beliefs and intentions.

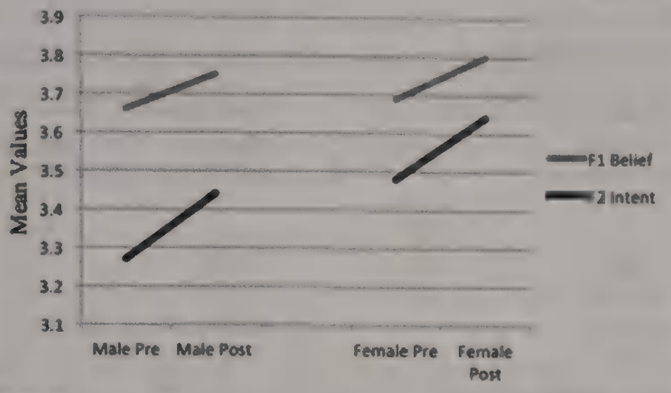

Figure 5. Comparison student male versus female pre-post for climate change beliefs and intentions.

the association between climate change intentions and climate change beliefs had increased to $r = .427$ ($p < .0005$) with 18% (.427 × .427) of their variance in common. This represents an increase from a moderate positive relationship between beliefs and intentions before MSOSW activities, to a large positive relationship after MSOSW activities.

As shown in Table 5, at the time of the pretest for comparison students, climate change beliefs and intentions were significantly correlated ($r = .312$, $p < .0005$) but the strength of the relationship was moderate (Cohen, 1988; Lenhard & Lenhard, 2016) with 10% (.312 × .312) of their variance in common. Also shown in Table 5, for the comparison group at the time of the posttest, the strength of the association between climate change beliefs and climate change intentions remained almost identical at $r = .317$ ($p < .0005$) with 10% (.317 × .317) of their variance in common. The moderate strength of association between climate change beliefs and intentions remained unchanged for the MSOSW comparison group, from pre to post.

As an answer to Research Question # 3, these findings provide evidence that a moderate (Cohen, 1988) positive relationship exists between climate change beliefs and intentions among middle school-aged students across the United States in their normal school environments, and that hands-on energy monitoring activities such as those provided by the Middle Schoolers Out to Save the World project can strengthen the association between the two constructs, from 9 to 18% of their variance in common. This is an increase from a moderate relationship to a moderately large relationship according to guidelines provided by Cohen (1988) of .1 = small, .3 = moderate, and .5 = large for Pearson Product Moment correlation coefficients. This is important because according to the theory of planned behavior (Ajzen & Fishbein, 1980), positive attitudes lead to a higher intention to perform a related behavior. In this case, the related behavior is intent to take action to reduce climate change.

Discussion

One purpose of this study was to measure the changes in climate change beliefs and intentions for students who participated in a hands-on, active learning curriculum in which students studied climate change and used take-home meters to measure standby power. By measuring the amount of power that was being wasted when appliances were plugged in yet serving no useful function, students were able to conclude that they could individually as well as collectively make a difference in the amount of CO_2 in the atmosphere. This kind of project is likely to have the type of impact reported by Corner et al. (2015) regarding youth engagement in climate change issues, due to the treatment students directly and experientially engaging with an educational activity regarding climate change that leads to action.

This study sought to address the disconnect between knowledge and attitudes versus action reported by many researchers (Leiserowitz, 2006; Sundblad et al. 2007; Weber, 2010) by following the principles of the theory of planned behavior (Ajzen & Fishbein, 1980) in which positive attitudes lead to a higher intention to perform a related behavior. It is also an attempt to address the

Table 4
Correlation of Climate Change Belief and Intention Factors at Pretest and Posttest Time for Treatment Students

		Pretest		Posttest	
		Belief	Intentions	Belief	Intentions
Climate change belief factor	Pearson correlation	1	.307**	1	.427**
	Sig. (two-tailed)		.000		.000
	N	792	792	733	733
Climate change intentions factor	Pearson correlation	.307**	1	.427**	1
	Sig. (two-tailed)	.000		.000	
	N	792	792	733	733

Note. **Correlation is significant at the .01 level (two-tailed).

Table 5
Correlation of Climate Change Beliefs and Intentions at Pretest and Posttest Time for Comparison Students

		Pretest		Posttest	
		Beliefs	Intentions	Beliefs	Intentions
Climate change belief factor	Pearson correlation	1	.312**	1	.317**
	Sig. (two-tailed)		.000		.000
	N	508	508	521	521
Climate change intentions factor	Pearson correlation	.312**	1	.317**	1
	Sig. (two-tailed)	.000		.000	
	N	508	508	521	521

Note. **Correlation is significant at the .01 level (two-tailed).

suggestion that individuals need specific experiences in ways they can realize change in order to take action (Wolf & Moser, 2011). In a study related to beliefs, Pajares (1992) concluded that "beliefs are the best indicators of the decisions individuals make throughout their lives" (p. 307). Dewey's "learning by doing and experiencing" principles were applied in this study to use hands-on learning curriculum to engage students in finding solutions to real-world problems to which they have experienced success in participating in making changes. Changing beliefs through hands-on real-world activities can likely change the intentions and behaviors of students who come to the conclusion that they can make a difference as an individual as well as a collective group. Recording a measurable increase in level of intentions for a specific group due to increase in beliefs may require waiting some time. For example, one school year cycle may be a reasonable amount of time lag to expect that strengthened beliefs in climate change might migrate through a learner's affective, reinforcing, and directive behavioral repertoire (Staats, 1975) to result in a measurable increase in intentions. Additional research is needed in this area.

Table 6
Comparison of Cohen's d *and Pearson* r *Effect Sizes*

d	*r*	Cohen (1988)	Hattie (2009)
.0	.00	No effect	Developmental effects
.1	.05		
.2	.10	Small effect	Teacher effects
.3	.15		
.4	.2		Zone of desired effects
.5	.24	Intermediate effect	
.6	.29		
.7	.33		
.8	.37	Large effect	
.9	.41		
>/= 1.0	.45		

Note. Adapted from Lenhard and Lenhard (2016).

The findings from this study with respect to gender indicate a stronger positive effect for girls than for boys regarding beliefs in climate change. This may be due to females feeling a stronger affinity for improving and making a difference in the world (Modi, Schoenberg, & Salmond, 2012). This type of learning environment provides interactions among students and provides a social learning context that females often prefer (Jacobs, Kuriloff, Andrus, & Cox, 2014) as well as providing connections between school and the real world.

Future research is planned to update the survey instrument with the additional three items and run factor analysis on various groups of participants. It is planned to use this data to compare the beliefs and intentions of the middle school students who were the focus of this study, to those of teachers and other STEM professionals.

In the broadest context, the research reported in this paper served as a test of whether the study of climate change beliefs and intentions can be anchored within existing behavioral theories used to predict an individuals' readiness to perform a given behavior (Ajzen, 1985). Initial indications are promising, as established through the confirmation of significant ($p < .05$) positive correlations between climate change intentions and beliefs, with stronger correlations emerging for students who have taken part in energy monitoring activities. However, correlation does not establish causation, so further research is needed in this area. It is possible that beliefs cause intentions, intentions cause beliefs, or something else causes both.

There are additional limitations to this study other than the causation issue reported in the previous paragraph. The subjects in this study were the students of teachers who volunteered to participate in a project to monitor unnecessarily wasted electricity and find ways to possibly reduce climate change. Therefore, these students' attitudes may have been influenced by their teachers, regardless of whether the teachers represented a treatment site or comparison site at the time of

acquisition of data. Also a large number of natural disasters occurred in the United States during the year (2015–2016) when this study took place. These events were in the public eye and may have influenced the treatment as well as comparison group students.

Summary and Conclusions

The findings presented in this paper provide evidence that curriculum projects with hands-on learning activities have a positive influence on climate change beliefs and intentions, and increase the strength of association between the two constructs. In this study, for the treatment group of MSOSW project students, climate change beliefs and intentions moved from a moderate positive relationship between beliefs and intentions before project activities, to a strong positive relationship after project activities. The comparison group remained largely unchanged. Both treatment and comparison groups became more positive in their climate change beliefs and intentions, pre to post, and in the case of beliefs the gains for treatment students were significantly larger than for the comparison students. Regarding gender differences in impact of treatment, females showed a larger positive change regarding climate change beliefs. These findings are similar to previous results reported by the authors on a different set of data (Christensen, Knezek, & Tyler-Wood, 2015) as well as findings from other similar studies (Redman & Redman, 2017).

Implications of these findings for educational practice include having students connect to real-world problems by participating in activities where they can make a small difference individually but a large improvement collectively. The impact of changes in beliefs and intentions for students in middle school grades is large considering that the impact can influence students for the rest of their lives, guiding them in the direction of making thoughtful decisions in their daily lives.

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Facilitating an Elementary Engineering Design Process Module

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STEM education in elementary school is guided by the understanding that engineering represents the application of science and math concepts to make life better for people. The Engineering Design Process (EDP) guides the application of creative solutions to problems. Helping teachers understand how to apply the EDP to create lessons develops a classroom where students are engaged in solving real world problems by applying the concepts they learn about science and mathematics. This article outlines a framework for developing such lessons and units, and discusses the underlying theory of systems thinking. A model lesson that uses this framework is discussed. Misconceptions regarding the EDP that children have displayed through this lesson and other design challenge lessons are highlighted. Through understanding these misconceptions, teachers can do a better job of helping students understand the system of ideas that helps engineers attack problems in the real world. Getting children ready for the 21st century requires a different outlook. Children need to tackle problems with a plan and not shrivel when at first, they fail. Seeing themselves as engineers will help more underrepresented students see engineering and other STEM fields as viable career options, which is our ultimate goal.

The value of an education heavily steeped in science, technology, engineering and math (STEM) has been advertised in the popular media as well as in education circles. Unfortunately, large-scale testing of school-aged children's science and mathematics understanding, like the Program for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study, have shown that the United States lags behind other industrialized nations (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005). Furthermore, the testing focus on reading and mathematics of the current political environment has edged out science and social studies from elementary classrooms (Walker, 2014). Careers in STEM fields are the ones that not only will be most lucrative, but will be necessary in the coming years and decades as more and more jobs float overseas, leaving the blue-collar workers in a chasm with regard to opportunities to hold a job to sustain a family. There will be an increasing need for STEM education to provide career paths and opportunities in a U.S. marketplace competing globally (Shilling, 2015). STEM careers represent the jobs that will be needed to continue to support innovation and change as we continue the fast forward into the 21st century (Trilling, Fadel, & Partnership for 21st Century Skills, 2009). The educational design espoused in the current project has been developed and rationalized via the genesis of research in STEM educational design and learning theory as guided by our review of the literature. Thus, the concurrent validity of our approach and the design we articulate is based upon our adoption of research-based STEM design frameworks technically evaluated for educational performance (Berland, 2013).

Engineering represents the application of science and mathematics concepts to make life better for people. This definition is the connecting glue for elementary-aged students and answers the inevitable question, "How am I ever going to use this?" Learning the concepts of mixtures and solutions in science and volume and capacity in mathematics ties directly to the human need for clean water. Communities around the world struggle with providing clean water for residents, including children. Research by the Girls Scouts Research Department indicates that in elementary school, girls are interested in STEM fields, and even can see themselves as holding a STEM career as adults (Modi, Schoenberg, & Salmond, 2012, p. 25). Even in middle school, girls indicate they like STEM fields, but other non-STEM career options become more prominent as they get older and work their way through high school. Health fields such as nursing and dental hygiene remain on young women's career paths, but engineering fields, mathematics fields, and even technology fall to the wayside as women choose fields where they perceive they will be able to help people and the world (Modi et al., 2012, pp. 25–27). Furthermore, developing an interest in STEM early is crucial. For males and females, according to Sadler et al., the interest students have upon entering high school is the best predictor of their career interests upon leaving high school (Sadler, Sonnert, Hazari, & Tai, 2012). The

challenge, then, is to help young women, along with all students of underrepresented populations, to see how STEM fields outside of the medical fields do help humanity and the environment.

Children's misconceptions about science concepts have theoretical underpinnings. Posner et al. propose two sets of knowledge, a child's naïve understandings and the adult's correct understanding about scientific phenomena. Children, when confronted with the disconnect abandon their naïve beliefs and reconceptualize their understanding according to the adult view (Posner, Strike, Hewson, & Hewson, 1982). Piaget even wrote about children experiencing disequilibrium when confronted with information that challenges their incorrect understandings, with the incorrect beliefs displaced by the new more mature understandings (Piaget, 1977). We will discuss children's misconceptions that were discovered as a result of using the lesson that is presented.

Helping elementary school teachers understand the pedagogy of how to connect engineering with science and mathematics and to use inquiry environments in new and creative ways is the first step to meet this challenge. Within this article, we share our conception of how to develop an understanding of the Engineering Design Process (EDP) along with a model for lesson design format teachers can use to create design challenges for their own units and lessons. Understanding the EDP will be developed through the model of an original lesson plan and the discussion of children's misconceptions of the elements of the design process.

EDP and STEM Education

According to Lammi, "Engineering design is more than the mere manipulation of numbers and the solving of scientific equations. The processes employed in engineering design encompass a broad variety of topics and fields of study" (Lammi, 2011, p. 3). In 1973, a professor at the University of Negev, Israel, created a short but revolutionary document on the "Design Process." Dr. Bernard Roth described an often-occurring problem, that engineers may come too quickly to a solution, only to find a better solution with further thought. In this document, he outlined a Design Process, describing how successful engineers go through a series of steps to critically think about the problem and as a result, create alternative approaches of solutions. "It is a series of steps through which the design passes before it is completed. *By making this a conscious process, the engineer can greatly improve his chances of arriving at a better solution*" (italics in original) (Roth, 1973, p. 4). This unassuming start is the foundation for the EDP. A critical outcome of facilitating student understanding of the EDP is that students engage in systems thinking where multiple viewpoints and elements are conceptualized toward problem solving (Silk & Schunn, 2008). The lesson model framework we propose incorporates a model for incorporating the EDP, while enabling students to overcome science misconceptions.

Table 1
Framework for an Engineering Design Challenge Module

Framework	
Lesson Title	Fill In
Literature opening	Title of book: Focus in the book:
Curriculum Standards	Source of Standards:
Science Inquiry	Lesson 1:
(attach lesson plans)	Lesson 2:
Introduction of Engineering Design Challenge	Design Challenge:
Sharing strategy	

Lesson Model for Engineering Design Challenges

Introducing children to the idea of engineering, the application of science and mathematics to make life better for people, helps to frame the lessons involved in an engineering design challenge module. Within the idea of a "module," teachers can structure science and math lessons that build concepts students can use to complete a design challenge. Ideally, a module would be a series of lessons on concepts students have some foundational knowledge. However, a module can be done in one day, or as a series of lessons culminating in a more complex design challenge. A design challenge is a "problem task" that students use the EDP to find a workable solution. The following will be a discussion to illustrate all of these ideas.

The Engineering Design Challenge Module

The Museum of Science, Boston has developed a series of engineering kits, called Engineering is Elementary (EiE) program kits, that models a very nice application of science and mathematics into a challenge for young engineers (http://eie.org/overview/engineering-design-process). Their model is one that is worthy of being a framework for the rest of us to use to develop our own curriculum (see Table 1). The use of this curriculum as a foundation for our model has a strong foundation in research. Student motivation, engagement, and enjoyment of science was seen to increase when instruction incorporated the EDP (Macalalag, Lowes, Guo, McKay, & McGrath, 2009). Further, teachers reported students of underrepresented and racial minorities in science showed improved performance (Engineering is Elementary Research

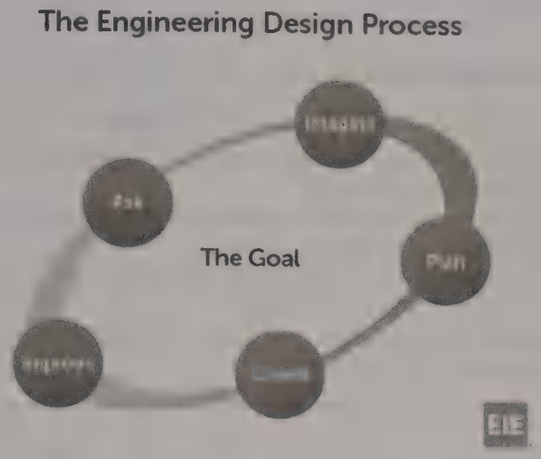

Figure 1. The EDP diagram from EIE.
Source: Teach Engineering Curriculum for K-12 Educators/Engineering Design Process 2003.

Department, 2014). Engaging students with a STEM curriculum illustrates how engineers apply science, technology and mathematics knowledge to solve problems (Sedberry, 2014), and integrated approaches to learning provide "more relevant, less fragmented, and more stimulating experience for learners" (Furner & Kumar, 2007, p.186). Finally, in research on teacher professional development by the Program for International Student Assessment (PISA), a significant correlation was found to exist between gains in science content knowledge and the number of engineering design activities students engaged in, showing a positive effect for the use of classroom curriculum with foundations in the EDP (Macalalag & Tirthali, 2010).

The framework starts with a literature link. The use of children's literature can help to provide a "real life" application for the concepts with the use of the story line. After a defining of the curriculum standards, the module moves into an inquiry activity with science and/or math concepts that will support the ultimate development of the design challenge. After discussion of the investigation, students are challenged to solve the problem that was introduced through the literature link. This is the actual jump into the EDP, illustrated in Figure 1.

EDP: How It Works

The EDP process, which takes on various forms depending on the publisher you choose, is the foundation of the process actually used by engineers. It consists of a series of steps that guide engineers as they solve problems (www.teachengineering.org/k12engineering/designprocess). One example from EiE is simple and an easy-to-follow model (Figure 2) (http://www.eie.org/). Almost always, the model is circular and does not indicate a particular starting point. This is because an engineer might jump into the process at

Figure 2. Examples of successful carrot Oscars.

any point, and although one generally goes through the steps in a clockwise fashion, even this is not a requirement. The model of a circle is important in that engineering is not a linear activity meant to reach the goal and be done, but a constant back and forth of questioning, creating and improving. Table 2 explains the steps in the design process, and illustrates each step with an example for a real product many of us use. A product may have a design flaw or something that needs improvement, which would lead an engineer to start with the improve step. This would be instead of starting at Ask, which would be where an engineer solving a problem for the first time would probably start. In the lesson that follows, the focus was specifically on the plan and create steps, getting children to not just jump into making something without thinking about it first, or without considering options. Teams had to submit their plan, complete with a materials list prior to getting materials, and being able to make a prototype.

Analyzing and Interpreting Data

A key component to the planning and implementation of this lesson is to provide children with an opportunity to collect data that can then be analyzed to use for creating a solution for the structured problem. This practice, Analyzing and Interpreting Data, is one of the key components of the NGSS Science and Engineering Practices (NGSS, 2013; Willard, 2014). Children in primary grades should be able to

Table 2
Engineering Design Process (EDP)

EDP Step	Definition	Example
Improve	A product has an aspect that is problematic.	The travel cup lid has a vent hole that drips coffee on the floor of the car.
Ask	Define an engineering type of problem.	How could the hole be closed so drips don't leak out?
Imagine	Brainstorm ideas.	Come up with several ideas.
Plan	Choose the best idea and sketch it out, make a list of materials, etc.	Develop the idea of putting the hole in the slide that closes the drinking hole.
Create	Make a prototype.	Make the prototype and test it—it works!
Improve	Is there a problem?	My car carpet is drip free.

test according to an attribute—in this case, does it float?—then record the data into a chart (Willard, 2014, p. 53). Teachers should be able to identify this step as inquiry-based and constructivist in nature, creating an experience for students to be hands-on with the available materials, testing their floating qualities, and proving for themselves what floats and what does not along with potential reasons why. The goal is then to see if children will use the data they gathered in creating solutions for the problem.

Systems Thinking

In systems thinking, the individual parts are identified as well as the understanding of how the parts work together to make the whole. "Analysis and design of systems is central to the work of engineers as they seek to modify their surroundings for particular purposes." (Silk & Schunn, 2008, p. 6). Lehrer and Schauble (1998) conducted interviews of second- through fifth-graders about gear systems that served a purpose, like in an egg beater or a bicycle, and gear systems that served no actual purpose. They found that second-graders had difficulty being able to connect the elemental parts of the system together to understand how the gears were working together. Fifth-graders were able to create "causal chains" of three or more parts, indicating a clearer understanding of how parts work together. However, both groups were better able to identify components that worked together when there was an action as an end result. For example, younger children were able to understand the purpose of the gears, and the teeth on them that interconnect that leads to the spinning of the beaters to whisk the eggs (National Academy of Engineering and National Research Council, 2009, p. 123). The NGSS Standards include Systems and System Models as a Crosscutting Concept (Willard, 2014, p. 53). For K–2 students, this involves being able to describe an object or organisms in terms of their parts, as well as being able to identify the parts that work together in the natural world as well as the designed world (Willard, 2014, p. 59). Engineering design challenges often ask students, even young students like the second-graders in the following example, to think about using materials and how materials work together to solve the problem.

Sample Teaching Module

Literature Opening

Using the Framework in Figure 1, we will outline a module created for second graders (Table 3) To start, we used the book, *The Hallo-Wiener*, by Dav Pilkey (1995). This is a story of dogs Trick-or-treating who get scared by costumed cats and run into a pond in fear for their lives. The underdog hero of the story is a dachshund, Oscar, who is wearing the embarrassing hotdog costume made by his mom. He hears the cries for help and as he waddles up he notices cat feet under the frightful costume. Oscar rips the costume off and the cats run away in fear. He jumps into the pond and swims to his friends who climb aboard his floating hot dog costume and Oscar becomes the hero of the night. In this module, we focused on the fact that Oscar's costume floats, and what do we know about why things float and sink. Although there is a serious breach of anthropomorphism, obviously, dogs do not dress up and go trick-or-treating, it is such a serious walk into fantasy that children see it as such. The further reason to use it is that in one of the most common basal reading series there is an author study highlighting Dav Pilkey, the author of the *Dumb Bunnies* series, *Captain Underpants* series, and other such favorites as *Dog Breath*, *Kat Kong*, and *Dogzilla*. The use of *The Hallow-Wiener* can tie together class work as well as entice teachers to use the engineering module.

In recent years, there has been an increase in the number of STEM, engineering and science focused books for children. Andrea Beaty has several, such as *Rosie Revere, Engineer* (Beaty, 2013) and *Iggy Peck, Architect* (Beaty, 2007). Other books introduce children to others who have worked to create something new, or used creativity to solve

Table 3
Model of the Framework in Action

Title	Oscar Floats!
Literature opening	Title of book: *The Hallo-Wiener* (D. Pilkey) Focus in the book: Oscar's costume floats
Curriculum standards	**Source of Standards**: *NGSS: Science and Engineering Practices* **Practice 2**: Develop a simple model based on evidence to represent a proposed object or tool. **Practice 3**: Plan and conduct an investigation collaboratively to produce data to serve as the basis for evidence to answer a question, and Make observations of a proposed solution to determine if it meets the goal. **Practice 4**: Record information and, analyze data from tests of an object or tool to determine if it works as intended.
Science inquiry (attach lesson plans)	Lesson 1: Inquiry: Test sample materials to determine floatability Lesson 2: n/a
Introduction of Engineering Design Challenge	Design Challenge: Design a "costume" for Oscar that will help him float.
Sharing strategy	Museum Walk

a problem. *The Boy Who Harnessed the Wind* (Kamkwamba & Mealer, 2012) and *Marvelous Mattie: How Margaret E. Knight Became an Inventor* (McCully, 2006) are both such books. Then others, like *The Hallo-Wiener*, are stories that inspire a design challenge. *Leo Cockroach, Toy Tester* (O'Malley, 1999) or *love flute* [*sic*] (Goble, 1992) may inspire challenges of flight or making musical instruments.

In planning a module, teachers would need to identify what the science concept is that grounds the design challenge inspired by the chosen book. Inquiry activities are readily available on almost any elementary science concept. Teachers may have textbook or kit resources in their schools, or they may use websites like www.sciencebuddies.org, info.collaborativeclassroom.org, or even STEMwise.education.olemiss.edu. The purpose of the inquiry needs to focus on developing the foundational understanding that children can then apply to the design challenge task to be successful.

The Standards

Referring to the framework, after the text has been identified, we can match the focus from the book to science or math standards. Although "floating and sinking" are no longer a specific part of the K–2 science curriculum, Process Standards from NGSS become the larger focus for this investigation. Practice 2 in the NGSS (2013) state that K–2 students need experiences in using diagrams and drawings as models to represent concrete design solutions (NGSS, Appendix F, p. 6). Specifically, students need to have opportunities to "Develop a simple model based on evidence to represent a proposed object or tool" (NGSS Appendix F, p. 6). This also ties in nicely with Practice 4, Analyzing and Interpreting Data, where students are to record information to identify patterns in the natural and designed world to solve problems. Students should be presented with opportunities to gather data to analyze and use in the construction of the solution strategy. Finally, students should "analyze data from tests to determine if it works as intended" (NGSS Appendix F, p. 9).

These process standards fit into the aspects of the EDP in this teaching module in the PLAN stage with "creating a model" (in this case a drawing/sketch), CREATE and IMPROVE steps. When students are planning, we want to see if they use the data from their prior tests of materials, analyzing materials that would be best and including them into the plan? Do the materials used work together as a system? Do the parts work together to make a workable solution? When they create, does it have anything to do with their plan? After they test it, do they understand that if Oscar sinks, that this is a failed test and they have to go back through the process to improve it? Do they work together and value the input of others? These questions help to focus adults as well, to keep the purposes of engineering design as the important elements.

Science Inquiry

The inquiry portion of this lesson focuses on understanding attributes of different materials, and what attributes are key to making them float. After reading the story to the class the teacher asks, "What could have been in Oscar's costume to make him float?" She asks questions to challenge student thinking about what attributes are important to make the material float (e.g., color is not an attribute that contributes to it floating). She shows the class a bag of sample materials and explains how they are going to test materials. She draws an "H" chart on the board (a chart with three columns). In the first column, she writes above the column "sinks," and "gets soggy" over the middle column and "floats" over the final column. Students

are asked to draw this chart into their engineering journals before starting the tests. Each team gets a samples baggie (with small bubble wrap, foam, Styrofoam peanuts, cardboard, pompon, paper, newspaper, plastic from a grocery bag, paper clip, etc.) and a "tub" of water with about 3 inches of water in it (a gallon jug with the top cut off is a perfect size).

Students test the materials and organize their data in the chart. When groups are done, a class discussion yields a class chart listing all of the items from the baggies. They then look closely at the items that floated. What do they have in common? What can we say about all of them? This can end the session for the day.

The Engineering Design Challenge

The next lesson introduces the design challenge. A quick review of the story shows that Oscar's costume floated, allowing him to rescue his friends. Reviewing the class chart from the previous investigation, the teacher can present the design challenge. Each team gets a 1′5″ to 2″ chunk of carrot with toothpick legs to represent "costumed" Oscar (Figure 2). Again in their teams, students need to plan a costume that will float so that he can save his friends, using the materials they tested. Teams will need to draw a model of their plan, and make a list of materials they will need (they may have unlimited string to attach materials to Oscar since tape will not work well in water). In this plan, they need to decide why different materials are included; how will the materials work together as a system to make Oscar float?

When they have created the prototype (Figure 2), they can test it in the water tub. If it does not float, teams can go back to the imagine stage and revise their plan and prototype. Even if it does float, they can work to make it even better.

When teachers create the design challenge, it needs to be purposeful, even if the purpose is imaginative, and the materials to be used or other constraints are included in the directions. For example, students may "Use cereal boxes to create a flying machine for Leo to get back to the Waddatoy Toy Company," a potential challenge for the book *Leo, Cockroach...Toy Tester* (O'Malley, 1999). Or, they might "Use toilet paper rolls, string, tissue paper, and rubber bands to create a musical instrument to send a message to someone you care about," a potential challenge for the book *love flute* (Goble, 1992). Constraints are limits put on the design, for instance, the materials may be only those specified, or inventors have a specified amount of time. Still others might constrain some aspect of how students create the work, like team members may only use left hands, or maybe they cannot talk. For this challenge, the task itself was demanding enough for our second-graders that we did not add other constraints.

Sharing Strategy

Once all teams (or most) have been successful, which may take two lesson periods, students can do a "museum walk" and "tour" the other teams' work to look for similarities and differences. A class discussion, highlighting patterns of successful prototypes and how teams made materials work together in their designs wraps up the module.

Misconceptions About Phases of the EDP

As "simple" as the EDP looks, children do develop misconceptions about the steps and the model itself. It is helpful for teachers to be aware of the misconceptions in order to guide discussions and lessons in general to minimize misunderstandings. Understanding misconceptions can help teachers feel more confident because this knowledge can give teachers an idea of what kinds of wrong turns might occur as they try new lessons with their children. There is not one right way to solve a design challenge, and in fact there can be as many different solutions as children bring fresh ideas to challenges.

After doing this series of lessons with K–2-aged children multiple times, misconceptions about the EDP have emerged. These recurring examples were collected through years of experience and not through a scientifically rigorous process. However, they can still be helpful for teachers in guiding elementary-aged children to develop habits of mind to facilitate problem solving and systems thinking.

Problems can emerge at the onset with the ASK element. Children can have a difficult time identifying researchable questions. Kindergarten through second-grade students are also learning to identify declarative and interrogative sentences (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), so when asked to listen for a question that needs a solution, as in this story "How can Oscar save his friends?" they identify any question in the story. So, when Oscar's mom presented him with a hot dog costume, and the narrator asks, "And guess who was supposed to fit in the middle?" (Pilkey, 1995, p.10) second-graders hands wave in the air! They identified the sentence as a question, correctly for English Language Arts, but not a question requiring an engineering response. Even children who had attended a STEM school and talked about the EDP throughout first grade identified questions between characters as questions for the ASK part of the EDP.

The IMAGINE step in the EDP is for brainstorming multiple ways to solve the problem (eie.org). In this case, multiple strategies for addressing the carrot to make it float. Seven- and eight-year-olds generally jump to the drawing, making one drawing and not thinking beyond that one

option. Encourage students to draw two or more different ideas, and then pick the best idea.

This lesson does try to focus particularly on the making of the design sketches to show their thinking before they are able to get materials to create. What is interesting about initial plans, particularly with children who have not had much experience with the EDP is that they will choose materials that are unlikely to work, despite the prior experience with testing. Children who have not had many experiences with the EDP consistently do not utilize information from the previous investigation—even if the investigation just happened. Although the children from the STEM school were still having trouble identifying questions, they were often able to make the carrot float on the first try because they utilized materials from the "floats" list. Children who are new to this type of thinking sometimes need to be prompted with a question such as, "What kind of information on this chart might help you?"

Last are the CREATE and IMPROVE steps. The idea of the lesson is to create their prototype, test it to see if it floats, and then if it does not, see if they can improve their design and try again. In an adult way of thinking this is a circular process, representing a failed trial, and a revision of the original design, leading to a new trial. In multiple times that this lesson was used, we hoped to document this process by having students call one of us over to take a picture or video record the trial, then come back and see the new trial, repeating for each trial until they were able to get it to float. Consistently with children new to the EDP, they do not call us over when they do test one. For the first several times doing this I thought they were not clear on the directions or that they got excited and forgot. I have worked to give the directions closer to the trials, made announcements, and had more adults ready to record. Yet there is a bigger issue: children see this process as linear instead of circular. Instead of being able to go back and repeat steps, they see the task as a linear function: make the carrot float. In watching one particular class, when I was able to catch a group try their first prototype, I realized that after they put it in and it did not float, it got snatched out of the water and the group's (unofficial) leader immediately went about making changes. But there was not a discussion, a review of their plan, or even a new plan made or discussed. It was not until they got it to float, that an adult would be called over. The task was completed.

Conclusions

Science teacher efficacy has been demonstrated to be a key index for measuring science teaching practice as those teachers who feel empowered by knowing both methodology and science content tend to teach inquiry-based science in classrooms (Liang & Richardson, 2009). Thus, both knowing content and feeling confident and motivated to teach hands-on science with designs that call for students conducting experiments is dependent upon a teachers' self-beliefs about their science knowledge. Lack of science efficacy leads to teacher avoidance of inquiry-based science. As prevalent as this problem is with elementary teacher, the lack of teachers' engineering efficacy is even greater.

In order to support teachers and students in inquiry-based science advocated in the current article, we suggest a review of Mott, Chessin, Sumrall, Rutherford, and Moore (2011) as the assessment of students' science products generated in inquiry-based science experiences can be supported with rubrics that align teacher-student and student-student discussions with science objectives. See the rubric online at: http://ojs.jstem.org/index.php?journal=JSTEM&page=index.

EDP "misconceptions" are important to understand. Students come to school with cultural and age-related conceptions about processes (Bransford, Brown, & Cocking, 1999). Teachers need to grasp the fact that this is an underpinning of understanding that teachers should not find fault for but realize that children need exposure to these alternate models of thinking. Particularly as we prepare more children to be college and career ready, children will only learn to persevere, for instance, if they understand that they have failed many times in their lives, and that it is not a bad thing, but you keep going and trying new strategies until you reach success. Or that the means to an end is multiple steps along the way, and that the end may not necessarily be the end, it may just start another question, such as How can we create lessons to dispel misconceptions about using the EDP? Teachers can find the EDP a useful thinking process that weaves throughout learning.

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Examining Recruitment and Retention Factors for Minority STEM Majors Through a Stereotype Threat Lens

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Prior research regarding minorities in the science, technology, engineering, and mathematics (STEM) fields indicated that the factors of peer support and participation in STEM-related activities contributed positively to minority students' recruitment and retention in these fields. Utilizing stereotype threat as a conceptual framework, this qualitative case study investigated the contribution of these factors by examining the experiences of minority students majoring in a STEM field. Data analyzed through a deductive approach indicated that a stereotype threat adversely affecting retention and recruitment in STEM could be either minimized or maximized by peer support, in the form of educator or familial influence, and participation in STEM-related activities through internships and the application of STEM-related knowledge. Findings from this research may inform higher education institutional practices involving the recruitment and retention of students to the STEM fields from minority populations.

There is a need to increase the number of minority students who major in a science, technology, engineering, or mathematics (STEM) fields. Historically, white males have dominated the population of professionals within these fields. Diversification of individuals within STEM careers can contribute new insights, viewpoints, and interpretations that aid the advancement of those fields in various ways. Therefore, increasing the strength of the various sections of the STEM pipeline (e.g., recruitment and retention) in higher education should be a goal. The STEM fields have historically seen little participation from minorities, although the numbers from this group are slowly increasing (Huang, Taddeuse, Walter, Samuel, & Washington, 2000, p. 3) and institutions of higher education have made great strides in attempts to recruit and retain STEM minority majors. In addition, STEM industries have become the fastest-growing sections of the economy and job growth (Vilorio, 2014). As the population of the United States continues to increase and become more racially and ethnically diverse, the demand to fill those jobs can be partially satisfied through an increase of the number of minorities entering the STEM fields. The United States Department of Education (2016) predicted that in the years following 2014, the minority student population in public elementary and secondary schools would become the new majority. Therefore, recruiting and retention efforts are imperative to ensure that more minority students pursue opportunities in STEM fields in K–16 and that their numbers increase at all levels of education.

Research has been conducted in order to determine those factors that contribute to an individual's initial decision to major in a STEM field, as well as persist in the field (Beasley & Fischer, 2012; Cole & Espinoza, 2008; Griffith, 2010). However, much more information is needed in order to close the gaps in the number of STEM professionals from minority populations. Great insight can be directly gained from minority students who have currently chosen a major in higher education in one of these fields. Through the interpretations of student experiences, knowledge can be expanded in order to formulate retention strategies for increasing the number of minorities that persist in the STEM fields. Still unknown are definitive factors that contribute to persistence in STEM fields; therefore, no focus has been placed on particular strategies that can improve this persistence (Griffith, 2010). This research project sought to uncover those features that contributed to minority student's perseverance, retention, in the STEM field based on the experiences of current minority STEM majors. Specifically, this study sought to answer the following questions: Why do minority students choose to pursue a degree in a STEM field? and What factors contribute to minority student's persistence for studying in their respective field?

Implications for practice from this study comprise the development of recruitment strategies to increase the number of minority students who pursue a degree in a STEM field. If there are specific experiences of minority students that allow for increased engagement in the STEM fields, then replicating such experiences would be desired. Researchers have noted far fewer minority students pursuing bachelor's degrees in general (Griffith, 2010), which has affected the number of minority students receiving bachelor's degrees in a STEM field. Through interviewing minority STEM students, this study aimed to

uncover and document those experiences that contributed to students' engagement and persistence in a STEM field.

Terminology

Ever since Judith Ramaley, former director of the National Science Foundation's Education and Human Resources Division, defined STEM education (Teaching Institute for Excellence in STEM, 2010), various professional organizations, institutions, and researchers have voiced opposing opinions concerning clear categorizations of the specific disciplines that compose STEM (Koonce, Zhou, & Anderson, 2011). For purposes of this study, the disciplines classified as STEM will be those described by Koonce et al. (2011) as "high-frequency STEM education fields" (p. 8), or the general fields of mathematics, physics, chemistry, biology, computer science, and engineering.

The definition of the term minority utilized in this study corresponds to those individuals who identify as Asian American or Pacific Islander, African American, Hispanic, or Native American. In concert with prior research by Museus, Palmer, Davis, and Maramba (2011), this study has not included mixed race individuals as defined by governmental agencies (U.S. Census Bureau, 2004a, 2004b). Furthermore, this particular research study utilized a general approach to minority membership in order to allow some freedom in the representation of the data as a wide-ranging study on minority STEM majors.

Literature Review

Prior research regarding minority students have sought to uncover those factors that contribute to persistence and retention, and research specific to minority STEM majors has pursued similar themes (Chang, Sharkness, Hurtado, & Newman, 2014; Cole & Espinoza, 2008; MacPhee, Farro, & Canetto, 2013). While very few factors have been singled out as contributing to the retention or attrition of minority students pursing STEM majors, agreement has been found that more study and evaluation is merited. Griffith (2010) maintained that increasing the number of students who choose to pursue a degree in a STEM field should be a priority. The reduced number of minorities in STEM fields have been identified as a continuing critical problem in the United States (Beasley & Fischer, 2012). The Higher Education Research Institute (2010) found that of those students from a minority background that did chose to major in a STEM field, only 16% finished their degree within five years. Therefore, determining what factors contribute to students' interest in STEM has and will continue to be of great concern.

A Case for Qualitative Methods

Most of the research involving recruitment and retention of minority students in STEM education has employed quantitative techniques (Chang et al., 2014; Cole & Espinoza, 2008; Hurtado et al., 2007; MacPhee et al., 2013). Studies involving qualitative research methods with regard to minorities in STEM education are necessary in order to provide information not found in quantitative studies. In fact, Museus et al. (2011), in their research on the current literature regarding those factors that contribute to minority STEM success, conveyed "the importance of moving beyond simplistic aggregated analyses to more complex understandings of the experiences and outcomes for various ethnic groups" (p. 20). Erlandson, Harris, Skipper, and Allen (1993) make a case for qualitative research by creatively portraying the rich expressive data that can result from naturalistic inquiry.

Relatively few qualitative research methods studies have been conducted to probe those factors that contribute to recruitment and retention of minority students in STEM (Palmer, Maramba, & Elon, 2011). Additionally, during the review of the literature, only one mixed methods study was found that involved qualitative research methods and all categories of minority STEM majors (Buschor, Berweger, Frei, & Kappler, 2014). Buschor et al. (2014) studied stereotype threat as a factor of STEM success. The authors included women in the study, another underrepresented STEM group.

Due to the limited amounts of qualitative research involving minority STEM majors, the study conducted by Palmer et al. (2011) served as the primary foundation for this study. This was predominantly due to these researchers utilization of the qualitative research method of interviews to determine those factors that contributed to the recruitment and persistence of minority STEM majors. Palmer et al. (2011) noted three themes that contributed to continuation and persistence in the STEM field. Peer support, participation in STEM-related activities, and strong secondary and elementary preparation influenced minority students' success in completion of a STEM field degree (Palmer et al., 2011). Results from the present study may corroborate these themes or uncover other various factors that may impact recruitment and persistence of minority STEM majors.

Recruitment and Retention Factors

The review of the literature also indicated that self-concept and self-efficacy affected the success of a minority student in the STEM fields (MacPhee et al., 2013), however, the volume of research in those areas was somewhat limited (Leslie, McClure, & Oaxaca, 2014).

MacPhee et al. (2013) found that students who persisted in a STEM field until graduation had higher levels of academic self efficacy, provided that these students also participated in a mentorship program. MacPhee et al.'s (2013) research asserted that retention of minority students in a STEM field was maintained through the use of mentorship programs.

Other researchers have quantitatively determined additional factors that contributed to the recruitment and retention of minority STEM majors. For example, Cole and Espinoza (2008) analyzed college grade point averages (GPA's) for minority students studying in a STEM field and whether or not particular environmental variables such as attending diversity related functions, student-faculty interactions, and negative feedback about academic work significantly contributed to GPA. Results from Cole and Espinoza's (2008) study designated that attending diversity functions was statistically significant for lowering a minority student's GPA. This finding contradicted previous research suggesting that attending events on campus that promote diversity would positively impact educational endeavors (Montelongo, 2003).

In addition, Hurtado, Carter, and Spuler (1996) documented that familial support and maintaining familial bonds were valuable aspects to transition and successful completion of a college degree. Gloria, Castellanos, and Rosales (2005) found that a perceived positive university environment, increased cultural congruity, and a decrease in the number of perceived barriers were related to persistence in higher education for minority Hispanic students. To summarize, similar research conducted with regard to minority STEM students found that the environment of the University and the respective STEM departments can have a great effect on the decision of a minority student to major in the STEM field (Griffith, 2010).

Conceptual Framework

Stereotype threat has been identified as one of the factors that may contribute to the retention and recruitment of minority students in a STEM field (Museus et al., 2011). Stereotype threat is the self-perceived concept and the apprehension caused by the idea that engaging in certain behaviors may confirm negative attributes commonly associated with minority group membership (Steele, 1997; Steele & Aronson, 1995). Stereotype threat was first described by Steele and Aronson (1995) in their study of how this socio-psychological notion affected the intellectual performance of African Americans. Steele and Aronson assigned African American and White students of similar intellectual abilities to two different groups. For one group a stereotype threat was introduced while the other served as a control. Each group was given a diagnostic exam of intellectual ability. Steele and Aronson found that academic performance of the African American students was significantly lower than their White counterparts when a stereotype threat was perceived. Since the inception of stereotype threat, other research has demonstrated the applicability of this idea to other groups often associated with a stereotype, such as females in mathematics (Inzlicht & Ben-Zeev, 2000) and students from low socioeconomic status (Croizet & Claire, 1998).

Stereotype threat was also known to be present when a particular minority group experiences the pressure to "disconfirm" the pertinent stereotype (Fogliati & Bussey, 2013). Furthermore, stereotype threat can describe the deficit in performance over several domains among minority groups (Beasley & Fischer, 2012). Results of these studies conveyed that cognizance of the stereotype was adequate enough to reduce minority intellectual performance (Steele & Aronson, 1995). There have been conflicting reports expressing that stereotype threat can cause motivational pressure, which can increase effort and success (Fogliati & Bussey, 2013; Jamieson & Harkins, 2007). For this study, stereotype threat will be used as a guide to understand what, if any, contributions this concept has on recruitment and retention of minority STEM majors.

Methodology

Lincoln and Guba (1985) noted, "objectivity exists when an appropriate methodology is employed" (p. 300). In order that objectivity be maintained, the qualitative research form of this investigation followed a case study design. Case studies encompass a complete description and an analysis of a particular instance contained within a certain context or boundary (Merriam, 1998). The instance of this study involved the experiences of minority STEM majors, while the context of the study occurs at a regional university in the Southwest. Case study emphasizes the complexities of a particular individual or program of special interest (Stake, 1995). While this study does not involve a specific program per se, the STEM fields and the issue of increasing the number of minorities within the fields lends itself to consideration as a program of special interest. Likewise, Merriam (1998) ascertained that a case study was the appropriate qualitative form when development of an understanding of the changing aspects of an individual or program was warranted.

Table 1
Classroom Observations

Number of Observations	Instructor's Field	Course Subject	Course Classification Level
3	Mechanical Engineering	Mechanical Engineering	Senior
3	Engineering Technology	General Engineering	Freshmen
3	Mathematics	General Calculus I	Sophomore
3	Mathematics	General Calculus II	Sophomore
3	Integrated Mathematics and Physics	General Physics	Freshmen

Participants

There were 57 undergraduate minority students who participated in this study. The students were studying in high-frequency STEM fields and from all classifications (freshmen, sophomore, junior, or senior); 50 answered surveys and 7 were interviewed. All participants were currently enrolled in an institution of higher education. A database of minority STEM students was generated at the institution and a recruiting email was sent to the students.

Data Sources

Observations, interviews, and document artifacts served as sources of data and assisted in the effort to triangulate the data for effectiveness and trustworthiness. Data collection began with observations of various higher education STEM classrooms and collection of document artifacts in the form of survey responses and seating charts. Semi-structure interviews were guided by themes and ideas that emerged from the observations and document artifacts.

Nonparticipant observations of minority students in a respective STEM major classroom were conducted throughout the course of a long semester. Observations documented the minority student's experience in the respective major course with emphasis on interactions with the professor and peers. Spradley (1979) maintained that selecting a social situation to conduct research should follow the selected criteria of simplicity, accessibility, frequently recurring activities, and participation. Table 1 depicts the various subjects and classifications for the classrooms in which observations occurred. Surveys as document artifacts were chosen due the fact that "artifacts can be technological devices ..., works of art, writing instruments, tools, and almost any other physical evidence" (Erlandson et al., 1993, p. 100). According to Merriam (1998) surveys are considered as researcher-generated artifacts and the "specific purpose for generating documents is to learn more about the situation, person, or event being investigated" (p. 119). Fifty students responded to the survey questions.

The final data source for this study consisted of interviews with minority STEM students. Interviews provided "access to the most complicated social and educational issues...abstractions based on concrete experiences of people" (Seidman, 2006, p. 1). Gubrium and Holstein (2003) posited that interviews allow for empirical data collection in a social aspect so that individuals may talk freely about their experiences and their lives. Seven interviews were conducted as part of this study, however interview data from only three students has been presented as part of the findings due to manuscript length constraints. These students (pseudonyms assigned) include Jose, a Hispanic senior computer science major; Anita, a Hispanic freshmen biology major; and Maria, a Hispanic sophomore biology major.

Data Analysis

Data analysis techniques for the interview and observation data consisted of color-coding techniques in order to develop categories and disaggregate core groupings that were relevant in providing answers to the research questions. Consistent with the deductive approach of this study, and based on prior research from Palmer et al. (2011), overarching themes of peer support and participation in STEM-related activities were first color coded. Each data source was color coded individually for these general categories.

Following this initial categorization, several iterations of coding were conducted using only the data that conformed to the general categories of peer support and participation in STEM-related activities. This allowed for saturation of the original categories and to allow new categories or subcategories to emerge (Erlandson et al., 1993, pp. 118–119). At this point, the new subcategories were compared between the different data sources in order to address the common categories established within the interviews, surveys, and observation notes. Subcategories were only included if they were present among all the three sources of data collection.

Trustworthiness

Trustworthiness was evaluated based on credibility, transferability, dependability, and confirmability. While every effort was made to ensure the trustworthiness of this

study Seidman (2006) argued that formulaic approaches were not necessary so long as respect and empathy for the issues of trustworthiness were maintained. Credibility was maintained through the use of member checks with interview participants and the interpretations formed as the interview unfolded. Every effort was made to assess the data for credibility and truthfulness, however Rossman and Rallis (2003) maintained that in qualitative studies, the data must be approached with humility and must be useful in answering the research questions while understanding that the data is in the perspective of the participant, not based on the researcher's notions. Credibility was also maintained through the use of an audit trail which consisted of line numbering in the interview transcriptions, observation notes, and surveys from which direct quotes can be traced.

Transferability can be extended as a qualitative research methodology that supports existing research. The data from this study also supported similar themes developed by Palmer et al. (2011), which further enhanced trustworthiness through transferability. Lincoln and Guba (1985) recommended that "anyone seeking to make a transfer is to accumulate empirical evidence about contextual similarity; the responsibility of the original investigator ends in providing sufficient descriptive data to make such similarity judgments possible" (p. 298). Reliability was achieved through the repeated means of data collection. Lincoln and Guba stated that reliability is achieved through demonstrated use of replication and inquiry within similar conditions. Triangulation through the use of document artifacts, interviews, and observations has contributed to the confirmability of this study.

Limitations

During the recruitment of faculty that provided opportunities for classroom observations for this study, communication was first initiated with the chairs of the departments of the mathematics, chemistry, and physics department; engineering and computer science department; and the life, earth, and environmental science department. Responses were received from the department chairs from the mathematics, chemistry, and physics and engineering and computer science department and as a result, faculty were recruited. No response was received from the life, earth, and environmental science department, although multiple methods of communication were attempted. This was considered a limitation of this study, as three of the seven interviews conducted originate from students in the biological sciences.

Time also served as a limitation of this study in two ways. The first being the time frame for collecting data through observations, interviews, and surveys. Permission for recruitment of participants for this study was not granted until mid-term of the long semester and, therefore, hastened the process of data collection to be completed before students completed final exams. Second, there was difficulty in coordinating observations of STEM classrooms due to teaching schedules of the faculty.

Although adding credibility and trustworthiness to the study, the researcher acknowledges personal experience as a STEM major from a minority background may be a possible limitation of this study. Personal experiences served as a primary catalyst for beginning this research study yet added a dimension of researcher bias, for the researcher approached the study with knowledge and personal experience of being a minority STEM major. To mitigate this particular bias, a deductive approach was utilized by considering the categories developed by the research conducted by Palmer et al. (2011). In this light, it must be noted that findings were developed based on prior research findings and not the researcher's bias.

Findings

Consistent with prior research findings from Palmer et al. (2011), the categories of peer support and participation in STEM-related activities were found to be present in all sources of data collection. Based on color-coding strategies in data analysis, these general categories were then broken into subcategories. Subcategories found to be present under the overarching category of peer support include educator and family influence. Subcategories present under the category of participation in STEM-related activities comprised internship experience and application of knowledge in a STEM field. All of the data, which fell into these subcategories, were then divided into whether or not the piece fell under the recruitment or retention of minority students within the STEM field. Table 2 depicts the major categories and subcategories developed within this study. The presentation of findings begins with the categories of peer support for recruitment, peer support for retention, participation in STEM-related activities as a recruitment tool, and finally participation in STEM-related activities for retention. For audit trail purposes, survey respondents were coded. For example, in the code S1Q1, the S1 indicates the survey respondent number and Q1 indicates the question responded to within the survey.

Peer Support for Recruitment

A subcategory most frequently present throughout the data was that of educator influence in recruitment of minority students within the STEM field. Using the surveys as document artifacts, 14 responses within the survey

Table 2
Categories and Subcategories of Recruitment and Retention

Recruitment	Retention
Peer Support	Peer Support
Educator Influence	Educator Influence
Family Influence	Peer Influence
Participation in STEM-Related Activities	Participation in STEM-Related Activities
Application of STEM Knowledge	Application of STEM Knowledge
Internship Experience	Internship Experience

specifically referred to a previous high school teacher as being a major inspiration or role model for pursuit of a STEM degree. Within the interviews, the notion of educator influence on the recruitment of minorities in STEM was also present. Anita mentioned a high school teacher conveying that this teacher "was really, she was really good. And really fun. And she never took shortcuts, she always showed us the long way but it was worth it" (Anita, pp. L184–185).

Findings of specific interest included three survey responses that referred to a high school math teacher as having an influential factor in the decision to major in a STEM field. This sentiment was echoed by Jose, a computer science major, who shared

> While I was in high school I was mostly leaning towards psychology and this math teacher of mine kept telling me that not to go for psychology, that I was good in math and that I should just go for something that had a lot of math. (Jose, pp. L226–229)

While much of the data indicated support through educator influence in a positive light, a senior Hispanic female biology major survey respondent indicated that although for her the educator influence was negative, it motivated her more to achieve a degree in her respective STEM field. She described the experience by stating that "I never passed a science class in high school. . . then got told I would be nothing in life by my advisor so I chose to prove them wrong and that's what inspired me" (S40Q10).

Family influence also contributed to recruitment of minorities within a STEM field course of study. When asked to describe what inspired the student to pursue a major in a STEM field, a junior Asian American female biology major responded to the survey question by stating that her mother served as inspiration "considering she was the only one to finish with a college degree when they came to America" (S16Q5). Other role models for minority STEM majors included family members who have also successfully pursued degrees in a STEM field. During the interview with Jose, he indicated a familial influence for recruitment to major in computer science. Jose stated that "an uncle of mine does software programming for like, um, libraries and stuff like that and he goes everywhere with that" (Jose, pp. L231–232).

Peer Support for Retention

Educator influence was also a frequently present factor contributing to the retention of minorities within a STEM field. A senior Hispanic female biology survey respondent, replied in the following manner to describe the best role model for completion of her STEM degree:

> Dr. Gershen (pseudonym)! He is an awesome teacher. He has taught me everything I know and has pushed me to realize what I really wanted to do in life but thought I was too dumb to do it. He completely inspires me and I could not thank him enough! (S28Q10)

Another survey respondent, a junior Native American male chemistry major, stated that when he needed assistance in his STEM courses, the professor's help was critical. "He has instructed me through my majors courses and I know that I would not have the accomplishments I do today without their guidance. I will be forever grateful" (S12Q9).

Educator influence as it related to retention was detected in the STEM major classroom observations. In an observation of a calculus I course, the interaction between a Hispanic male mathematics major and the instructor showed high engagement in the material by both parties. When debriefing the instructor, he commented that this student was always highly involved during the lecture and did well, even though the student himself had the idea that he was not doing well.

Peer influence in the form of tutors, classmates, and friends was another key category that appeared in the analysis of the data to support or produce barriers to STEM degree completion for minority students. In 23 out of the 50 surveys, respondents indicated that when assistance was needed in their respective STEM major courses, they often

sought out the help of a tutor or friends. One survey respondent, a freshman Hispanic female mechanical engineer, verbalized that one barrier to STEM degree completion was "being the only female in some classes, having people tell me I should switch majors when I slip up" (S14Q7). This same respondent went on to state the most frustrating thing about majoring in a STEM field at this university was "people who have this underlying assumption that im [sic] not as smart as my white male classmates" (S14Q8). This type of comment was reiterated in the survey responses from a sophomore Hispanic female mechanical engineering major who stated there were "Not many femals [sic] even less that are Hispanic" (S9Q), perceiving this was a barrier to her earning a STEM degree.

An interesting finding emerged from Maria's interview. She referred to a sense of community that evolved with other minority peers in her course of study. In order to receive assistance with the course material, Maria attended an organized study group led by a student paid through a university service. Until she was asked by the researcher to describe the other minority students majoring in biology, Maria was unaware that all of the students that attended the study session were minorities, including the student leader.

Peer support for retention was apparent in the observations of the STEM classrooms and was detected through minority students tending to group together, which occurred in all the classroom observations. This was especially evident in the seating choice of minorities attending the STEM classes. All of the STEM classrooms were organized into rows and columns for seating, with students having a choice of seat placement. When considering a 3 by 3 square of seating with a minority student seated in the middle seat of the middle row, 76% of such seating arrangements saw three or more minority students seated within the 3 by 3 square, in addition to the one seated in the middle.

STEM-Related Activities and Recruitment

Survey respondents indicated that STEM-related activities, such as starting a school robotics team, involvement in a science fair, membership in a welding team, and participating in an engineering club, contributed to the minority students' interest in continuation of study in a STEM field at the postsecondary level. One survey respondent, a freshman Hispanic female biology major, asserted the importance of STEM-related activities for recruitment by stating that "it was such an exhilarating experience to dissect animals. Apart from my awesome teacher, I also enjoyed our field trip to a medical school and seeing human cadavers" (S17Q6).

Internships as a form of participation in STEM-related activities also served as a recruiting tool for minority STEM majors. One survey respondent commented that an internship with a family health clinic inspired her to pursue a major in biology, as did another survey respondent who participated in an internship through the high school she attended. Anita referenced an internship experience in which she aided missionaries in providing medical care to individuals in a third world country. For Anita, the experience was the most influential factor in her life that affected her pursuit of a major in biology.

STEM-Related Activities and Retention

Application of knowledge in a STEM field through STEM-related activities was also categorized as a retention factor for minority STEM students at this university and served as a catalyst for majoring in a STEM field. For example, an African American computer science major referenced his love of technology as inspiration for choosing his major and stated that "I started thinking if I like engineering so much I should engineer something that I love, which computers was that something" (S21Q5).

Observation of the freshmen level engineering and the sophomore level calculus II class also confirmed this notion of application of knowledge as important to the retention of minority STEM students. Specifically, in these two classes the minority students were more engaged and involved in the particular STEM content as they were applying their knowledge through STEM-based activities. In the freshman-level engineering course, students were drawing and designing various objects using a compass, protractor, and straight edge. In the sophomore calculus II class, students were engaged in working problems that were real-world application based.

Discussion

Any discussion of the findings must begin with revisiting the conceptual framework's relationship to the categorical findings and the recruitment and retention of minority STEM majors. In reference to educator and peer support for recruitment, a stereotype threat adversely affecting recruitment in STEM could possibly be minimized or maximized through the influence of an educator or family member. This was apparent in the decision for Jose to major in computer science after having a teacher comment on his excellence in mathematics and despite his reluctance to major in a STEM field because of his self-doubt of being able to excel. Confirmation of this finding existed in the survey comments about the robotics teacher who pushed one student to "pursue those things which I was afraid of." These findings are consistent with prior research conducted by Rivardo, Rhodes, and Klein (2008), who found that stereotype threat was less likely to affect academic performance with a cultivating and personable environment.

Aronson et al. (1999) asserted that stereotype threat interference with performance does not require a history of negativity or feelings internalized relating to academic inferiority, but can solely originate from situational pressures.

Sackett, Hardison, and Cullen (2004) referred to the hypothesis that stereotype threat in a situation becomes striking when concerns about being judged with regard to a stereotype arise to adversely affect performance. Familial influence as a recruitment tool for minority STEM majors allowed for support in the form of a mentor, someone who has already achieved a STEM degree. This finding was evidenced throughout the surveys and interviews and also confirmed the findings of MacPhee et al. (2013) who uncovered that mentorship positively affected retention. Likewise, Museus et al. (2011) indicated in their review of the literature that parental involvement and support can facilitate the recruitment of minority students within a STEM field. Knowing others who have completed a STEM degree with success may allow for those stereotype threat concerns to be voiced and addressed by someone who may have had the same experiences. Good, Aronson, and Inzlicht's (2003) study determined that effects of stereotype threat can be reduced or eliminated when psychological interventions were applied.

Carr and Steele (2009) maintained that the burden of negative stereotypes about one's group interferes with one's capacity to adapt to new situations. Inability to adapt causes individuals to revert to attributes, attitudes, and situations familiar to them. As evidenced in all data sources, students within the STEM courses often conversed only with other minority students, studied with other minority students, and sat next to other minority students. Museus et al. (2011) suggested that minority students in general "feel increased susceptibility to being stereotyped, which could make them more vulnerable to stereotype threat" (p. 36). In the study conducted by Good et al. (2003), mentorship by college student peers was shown to minimize and in some instances eradicate stereotype threats. Minority students relying on themselves for peer support was evidenced and possibly provided a certain level of mentorship.

Therefore, in response to the first research question, findings from this study indicated that educator and familial influence as well as participation in STEM-related activities with respect to application of knowledge and internship experience affected recruitment of minority students to a STEM field. Within these categories and following from the data sources, educators have the most influence toward recruitment of minority STEM majors.

In regard to the second research question, factors that contributed to persistence and therefore retention of minority STEM majors were educator, classmate, peer, or tutor influence and participation in STEM activities involving the application of knowledge and internships. Strong factors for retention of minority students who pursue a STEM degree included the educator and peer influence.

Suggestions for future studies include exploring the conceptions that mathematics or mathematics teachers have regarding minority STEM recruitment and retention. Findings from this study indicated that this was a common subcategory not fully explored within the data collected and should be examined further. Other research studies should investigate what factors mathematics in general has on minority STEM recruitment and retention. Museus et al. (2011) contended that the literature suggested that African Americans and Hispanics were placed into remedial mathematics courses at disproportionate rates, which would affect their success in many STEM fields. Implications for practice from this study reiterate the idea of the impact and influence an educator has on the life of students, more specifically minority STEM students. Educators in the STEM fields must be made aware of their impact on students from these populations and efforts be made to inspire their decision to major in STEM. Moreover, instructors in higher education must also be aware of the impression they make with retaining these students once a STEM course of study is chosen.

Recommendations and Conclusion

As stated previously in the review of the literature, more research is necessary in order to increase the number of minority students who pursue a STEM field. Stereotype threat has been known to adversely affect academic performance (Aronson et al., 1999; Beasley & Fischer, 2012; Carr & Steele, 2009; Croizet & Claire, 1998; Fogliati & Bussey, 2013; Sackett et al., 2004; Steele, 1997; Steele & Aronson, 1995). However, Good et al. (2003) showed that this threat can be managed or eliminated as a factor affecting success in minority groups. Determining the factors that positively contribute to minority STEM major recruitment and retention, may allow for the elimination of stereotype threats that hinder academic success for minority students in the STEM fields. Increasing the numbers of minorities in STEM careers should be a goal of many institutions in order to diversify the number of professionals in these areas.

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Author's Note

Keywords: attitudes/beliefs, learning processes, culture, foundations, social/society.

PROBLEMS section

https://doi.org/10.1111/ssm.12263

Ted Eisenberg, *Section Editor*

This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to eisenbt@013.net. Solutions to previously stated problems can be seen at http://www.ssma.org/publications.

Solutions to the problems stated in this issue should be posted before March 15, 2018

• **5475**: *Proposed by Kenneth Korbin, New York, NY*

Given positive integers a, b, c, and d such that $\begin{cases} a+b=14\sqrt{ab-48}, \\ b+c=14\sqrt{bc-48}, \\ c+d=14\sqrt{cd-48}, \end{cases}$

with $a < b < c < d$. Express the values of b, c, and d in terms of a.

▪ **5476**: *Proposed by Ed Gray, Highland Beach, FL*

Find all triangles with integer area and perimeter that are numerically equal.

▪ **5477**: *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu-Sevrin, Mehedinti, Romania*

Compute:

$$L=\lim_{n\to\infty}\left(\ln n+\lim_{x\to 0}\frac{1-\sqrt{1+x^2}\sqrt[3]{1+x^2}\cdot\ldots\cdot\sqrt[n]{1+x^2}}{x^2}\right).$$

• **5478**: *Proposed by D. M. Batinetu-Giurgiu, "Matei Basarab" National Collge, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" Secondary School, Buzau, Romania*

Compute:

$$\int_0^{\pi/2}\cos^2 x\left(\sin x\sin^2\left(\frac{\pi}{2}\cos x\right)+\cos x\sin^2\left(\frac{\pi}{2}\sin x\right)\right)dx.$$

▪ **5479**: *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

Let $f:[0,1]\to\Re$ be a continuous convex function. Prove that

$$\frac{2}{5}\int_0^{1/3}f(t)dt+\frac{3}{10}\int_0^{2/3}f(t)dt\geq\frac{5}{8}\int_0^{8/15}f(t)dt.$$

• **5480**: *Proposed by Ovidiu Furdui, Technical University of Cluj-Napoca, Cluj-Napoca, Romania*

Let $n\geq 1$ be a nonnegative integer. Prove that in $C[0,2\pi]$

$$span\{1,\sin x,\sin(2x),\ldots,\sin(nx)\}=span\{1,\sin x,\sin^2 x,\ldots,\sin^n x\}$$

if and only if $n=1$.

We mention that $span\{v_1,v_2,\ldots,v_k\}=\sum_{j=1}^{k}a_jv_j$, $a_j\in\Re, j=1,\ldots,k$, denotes the set of all linear combinations with $v_1,v_2,\ldots,v_k$.

PROBLEMS section

https://doi.org/10.1111/ssm.12264

Ted Eisenberg, *Section Editor*

This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to eisenbt@013.net. Solutions to previously stated problems can be seen at http://www.ssma.org/publications.

Solutions to the problems stated in this issue should be posted before April 15, 2018

• **5481**: *Proposed by Kenneth Korbin, New York, NY*
A triangle with integer area has integer length sides $(3, x, x+1)$. Find five possible values of x with $x > 4$.

▪ **5482**: *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu-Severin, Mehedinti, Romania*
Prove that if n is a natural number then

$$\frac{(\tan 5^\circ)^n}{(\tan 4^\circ)^n+(\tan 3^\circ)^n}+\frac{(\tan 4^\circ)^n}{(\tan 3^\circ)^n+(\tan 2^\circ)^n}+\frac{(\tan 3^\circ)^n}{(\tan 2^\circ)^n+(\tan 1^\circ)^n}\geq\frac{3}{2}.$$

▪ **5483**: *Proposed by D. M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest and Neculai Stanciu, "George Emil Palade" School Buzău, Romania*
If $a, b > 0$, and $x \in \left(0, \frac{\pi}{2}\right)$ then show that
1. $(a+b)\cdot\frac{\sin x}{x}+\frac{2ab}{a+b}\cdot\frac{\tan x}{x}\geq\frac{6ab}{a+b}$.

2. $a\cdot\tan x+b\cdot\sin x > 2x\sqrt{ab}$.

▪ **5484**: *Proposed by Mohsen Soltanifar, Dalla Lana School of Public Health, University of Toronto, Canada*
Let X_1, X_2 be two continuous positive valued random variables on the real line with corresponding mean, median, and mode $\overline{x_1}$, $\tilde{x}_1$, $\hat{x}_1$ and $\overline{x_2}$, $\tilde{x}_2$, $\hat{x}_2$ respectively. Assume for their associated cumulative distribution functions (CDFs), we have

$$F_{X_1}(t) \leq F_{X_2}(t) \quad (t > 0).$$

Prove or give a counter example:
(a) $\overline{x_2} \leq \overline{x_1}$, (b) $\tilde{x}_2 \leq \tilde{x}_1$, (c) $\hat{x}_2 \leq \hat{x}_1$.

▪ **5485**: *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*
Let x, y, z be three positive real numbers. Show that

$$\prod_{\text{cyclic}}(2x+3y+z+1)\sum_{\text{cyclic}}(4x+2y+1)^{-3}\geq 3.$$

• **5486**: *Proposed by Ovidiu Furdui, Technical University of Cluj-Napoca, Cluj-Napoca, Romania*
Let $(x_n)_{n\geq 0}$ be the sequence defined by $x_0=0, x_1=1, x_2=1$ and $x_{n+3}=x_{n+2}+x_{n+1}+x_n+n$, $\forall n \geq 0$. Prove that the series $\sum_{n=1}^{\infty}\frac{x_n}{2^n}$ converges and find its sum.

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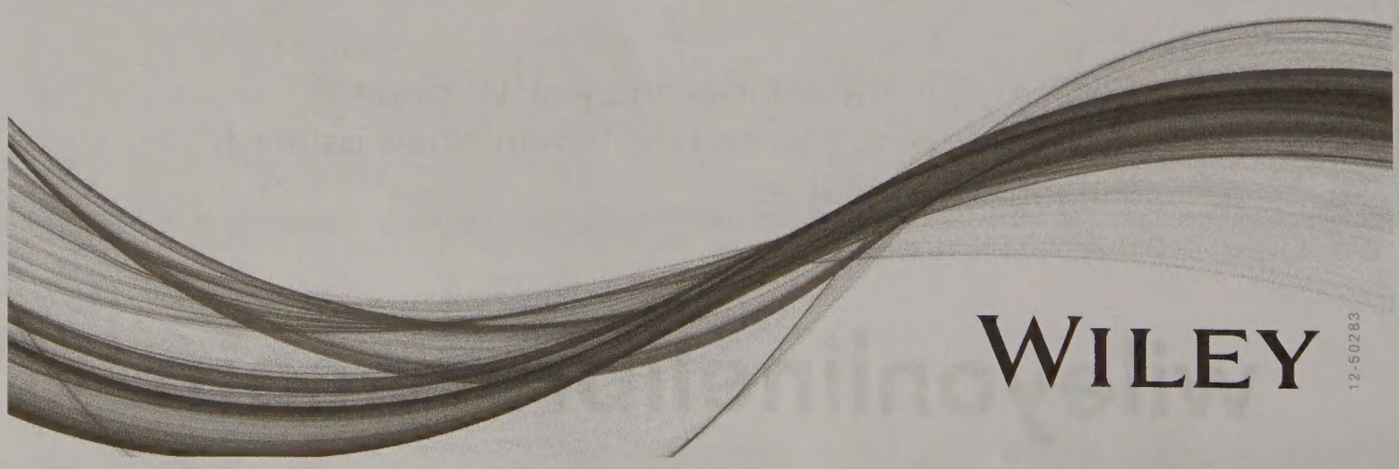

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SSM Publication Guidelines

Journal Office:
Dr. Carla C. Johnson, Editor
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Circulation Data:
Reader: K-graduate leaders and teacher educators
Frequency of issue: 8 times per year
Copies per issue: 3001-4000
Sponsorship: Professional organization

Publication Guidelines:
Manuscript length, 6-26 pages including references. In addition, up to 5 pages of tables/figures/appendices will be acceptable. Tables/figures should fit on no more than one page each.

Review Information:
Type of review: Blind review
No. of external reviewers: Three
No. of in-house reviewers: One
Time to review: 6-7 months
Acceptance rate: 20%
Reviewers' comments are returned to the author

Manuscript Topics:

Science, mathematics, and connections between science and mathematics for grades K-graduate and teacher education, especially science and mathematics articles that deal with assessment, attitudes, beliefs, curriculum, equity, research, translating research into practice, learning theory, alternative conceptions, philosophy and history of science, sociocultural issues, special populations, technology, nontraditional forms of instruction, and science/technology/society.

Manuscript Guidelines/Comments:

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The lead author must be a member of the School Science and Mathematics Association to submit the article and must remain a member in good standing at the time of publication of the paper. Membership information can be found at the Association web site www.ssma.org.

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